OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4763

Mechanics 3

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes

4763

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

1 **(a)** Two light elastic strings, each having natural length 2.15 m and stiffness $70\,\text{N m}^{-1}$, are attached to a particle P of mass 4.8 kg. The other ends of the strings are attached to fixed points A and B, which are 1.4 m apart at the same horizontal level. The particle P is placed 2.4 m vertically below the midpoint of AB, as shown in Fig. 1.

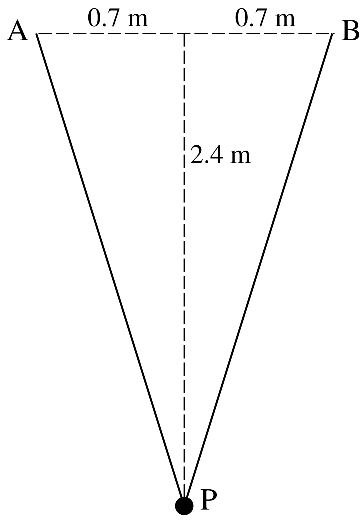

Fig. 1

(i) Show that P is in equilibrium in this position. **[6]**

(ii) Find the energy stored in the string AP. **[2]**

Starting in this equilibrium position, P is set in motion with initial velocity $3.5\,\text{m s}^{-1}$ vertically upwards. You are given that P first comes to instantaneous rest at a point C where the strings are slack.

(iii) Find the vertical height of C above the initial position of P. **[4]**

(b) **(i)** Write down the dimensions of force and stiffness (of a spring). **[2]**

A particle of mass m is performing oscillations with amplitude a on the end of a spring with stiffness k. The maximum speed v of the particle is given by $v = cm^{\alpha}k^{\beta}a^{\gamma}$, where c is a dimensionless constant.

(ii) Use dimensional analysis to find α, β and γ. **[4]**

2 A hollow hemisphere has internal radius 2.5 m and is fixed with its rim horizontal and uppermost. The centre of the hemisphere is O. A small ball B of mass 0.4 kg moves in contact with the smooth inside surface of the hemisphere.

At first, B is moving at constant speed in a horizontal circle with radius 1.5 m, as shown in Fig. 2.1.

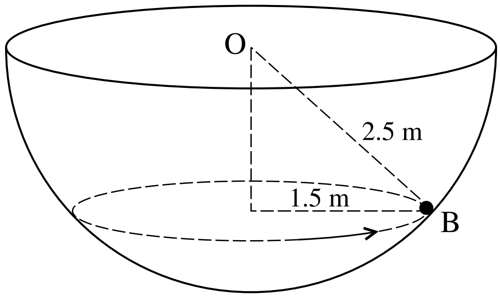

Fig. 2.1

(i) Find the normal reaction of the hemisphere on B. **[3]**

(ii) Find the speed of B. **[3]**

The ball B is now released from rest on the inside surface at a point on the same horizontal level as O. It then moves in part of a vertical circle with centre O and radius 2.5 m, as shown in Fig. 2.2.

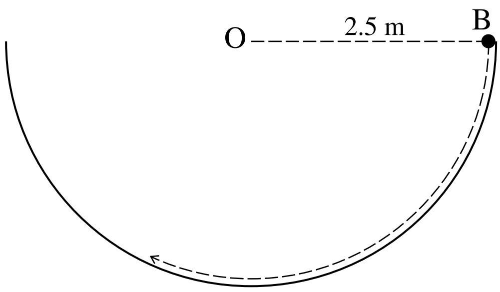

Fig. 2.2

(iii) Show that, when B is at its lowest point, the normal reaction is three times the weight of B. **[4]**

For an instant when the normal reaction is twice the weight of B, find

(iv) the speed of B, **[5]**

(v) the tangential component of the acceleration of B. **[3]**

3 In this question, give your answers in an exact form.

The region R_1 (shown in Fig. 3) is bounded by the x-axis, the lines $x = 1$ and $x = 5$, and the curve $y = \frac{1}{x}$ for $1 \leqslant x \leqslant 5$.

(i) A uniform solid of revolution is formed by rotating the region R_1 through 2π radians about the x-axis. Find the x-coordinate of the centre of mass of this solid. **[5]**

(ii) Find the coordinates of the centre of mass of a uniform lamina occupying the region R_1. **[7]**

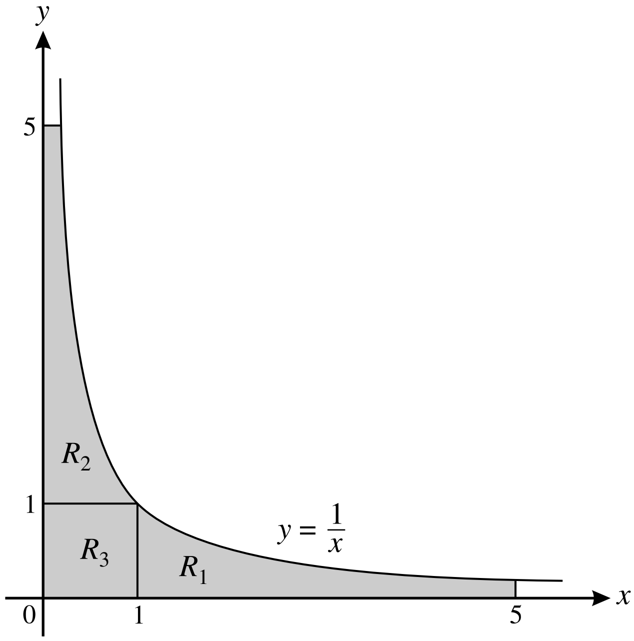

Fig. 3

The region R_2 is bounded by the y-axis, the lines $y = 1$ and $y = 5$, and the curve $y = \frac{1}{x}$ for $\frac{1}{5} \leqslant x \leqslant 1$. The region R_3 is the square with vertices (0, 0), (1, 0), (1, 1) and (0, 1).

(iii) Write down the coordinates of the centre of mass of a uniform lamina occupying the region R_2. **[2]**

(iv) Find the coordinates of the centre of mass of a uniform lamina occupying the region consisting of R_1, R_2 and R_3 (shown shaded in Fig. 3). **[4]**

4 A particle P is performing simple harmonic motion in a vertical line. At time t s, its displacement x m above a fixed point O is given by

$$x = A \sin \omega t + B \cos \omega t$$

where A, B and ω are constants.

(i) Show that the acceleration of P, in $\mathrm{m\,s^{-2}}$, is $-\omega^2 x$. **[3]**

When $t = 0$, P is 16 m *below* O, moving with velocity $7.5\ \mathrm{m\,s^{-1}}$ *upwards*, and has acceleration $1\ \mathrm{m\,s^{-2}}$ *upwards*.

(ii) Find the values of A, B and ω. **[4]**

(iii) Find the maximum displacement, the maximum speed, and the maximum acceleration of P. **[5]**

(iv) Find the speed and the direction of motion of P when $t = 15$. **[2]**

(v) Find the distance travelled by P between $t = 0$ and $t = 15$. **[4]**

GCE

Mathematics (MEI)

Advanced GCE **4763**

Mechanics 3

Mark Scheme for June 2010

1(a)(i)	$\text{AP} = \sqrt{2.4^2 + 0.7^2} = 2.5$ Tension $T = 70 \times 0.35 \quad (= 24.5)$ Resultant vertical force on P is $2T\cos\theta - mg$ $= 2 \times 24.5 \times \frac{2.4}{2.5} - 4.8 \times 9.8$ $= 47.04 - 47.04 = 0$ Hence P is in equilibrium	M1 A1 M1 B1 B1 E1	**6**	Attempting to resolve vertically For $T \times \frac{2.4}{2.5}$ (or $T\cos 16.3°$ *etc*) For 4.8×9.8 Correctly shown
(ii)	$\text{EE} = \frac{1}{2} \times 70 \times 0.35^2$ Elastic energy is 4.2875 J	M1 A1	**2**	(M0 for $\frac{1}{2} \times 70 \times 0.35$) *Note* If 70 is used as modulus instead of stiffness: (i) M1A0M1B1B1E0 (ii) M1 A1 for 1.99
(iii)	Initial $\text{KE} = \frac{1}{2} \times 4.8 \times 3.5^2$ By conservation of energy $4.8 \times 9.8h = 2 \times 4.2875 + \frac{1}{2} \times 4.8 \times 3.5^2$ $47.04h = 8.575 + 29.4$ Height is 0.807 m (3 sf)	B1 M1 F1 A1	**4**	Equation involving EE, KE and PE (A0 for 0.8) ft is $\frac{2 \times \text{(ii)} + 29.4}{47.04}$
(b)(i)	$[\text{ Force }] = \text{M L T}^{-2}$ $[\text{ Stiffness }] = \text{M T}^{-2}$	B1 B1	**2**	*Deduct 1 mark if units are used*
(ii)	$\text{L T}^{-1} = \text{M}^{\alpha} (\text{M T}^{-2})^{\beta} \text{L}^{\gamma}$ $\gamma = 1$ $\beta = \frac{1}{2}$ $0 = \alpha + \beta$ $\alpha = -\frac{1}{2}$	B1 B1 M1 A1	**4**	Considering powers of M *When [Stiffness] is wrong in (i), allow all marks ft provided the work is comparable and answers are non-zero*

2 (i)	$R\cos\theta = mg$ [θ is angle between OB and vertical] $R \times 0.8 = 0.4 \times 9.8$ Normal reaction is 4.9 N	M1 A1 A1 **3**	Resolving vertically
(ii)	$R\sin\theta = m\frac{v^2}{r}$ $4.9 \times 0.6 = 0.4 \times \frac{v^2}{1.5}$ $v^2 = 11.025$ Speed is $3.32\ \mathrm{m\,s^{-1}}$ (3 sf)	M1 A1 A1 **3**	For acceleration $\frac{v^2}{r}$ or $r\omega^2$ or $4.9 \times 0.6 = 0.4 \times 1.5\omega^2$ ft is $1.5\sqrt{R}$
(iii)	By conservation of energy $\frac{1}{2}mu^2 = mg \times 2.5$ $u^2 = 5g$ ($u = 7$) $R - mg = m \times \frac{u^2}{2.5}$ $R - mg = 2mg$ $R = 3mg$	M1 A1 M1 E1 **4**	Equation involving KE and PE Vertical equation of motion (must have three terms) Correctly shown or $R = 11.76$ and $3 \times 0.4 \times 9.8 = 11.76$
(iv) **(v)**	$\frac{1}{2}mv^2 = mg \times 2.5\cos\theta$ $v^2 = 5g\cos\theta$ $R - mg\cos\theta = m \times \frac{v^2}{2.5}$ When $R = 2mg$ ($= 7.84$), $2mg - mg\cos\theta = \frac{mv^2}{2.5}$ $2mg - \frac{mv^2}{5} = \frac{mv^2}{2.5}$ $7.84 - 0.08v^2 = 0.16v^2$ $v^2 = \frac{98}{3}$ Speed is $5.72\ \mathrm{m\,s^{-1}}$ (3 sf) $\cos\theta = \frac{v^2}{5g} = \frac{2}{3}$ ($\theta = 48.2°$ or 0.841 rad) Tangential acceleration is $g\sin\theta$ Tangential acceleration is $7.30\ \mathrm{m\,s^{-2}}$ (3 sf)	M1 A1 M1 M1 M1 A1 M1 A1 **8**	*Mark (iv) and (v) as one part* Equation involving KE, PE and an angle (θ is angle with vertical) [$\frac{1}{2}mv^2 = mgh$ can earn M1A1, but only if $\cos\theta = h/2.5$ appears somewhere] Equation of motion towards O (must have three terms, and the weight must be resolved) Obtaining an equation for v Obtaining an equation for θ *These two marks are each dependent on M1M1 above* [$g\sin\theta$ in isolation only earns M1 if the angle θ is clearly indicated]

3 (i)	Volume is $\int_1^5 \pi\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	π may be omitted throughout *Limits not required*
	$= \pi\left[-\frac{1}{x}\right]_1^5 \quad (=\frac{4}{5}\pi)$	A1	For $-\frac{1}{x}$
	$\int \pi x y^2\,\mathrm{d}x = \int_1^5 \pi x\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	*Limits not required*
	$= \pi\left[\ln x\right]_1^5 \quad (=\pi\ln 5)$	A1	For $\ln x$
	$\bar{x} = \frac{\pi\ln 5}{\frac{4}{5}\pi} = \frac{5\ln 5}{4}$ *(2.012)*	A1 **5**	*SR* If exact answers are not seen, deduct only the first A1 affected
(ii)	Area is $\int_1^5 \frac{1}{x}\mathrm{d}x$	M1	*Limits not required*
	$= \left[\ln x\right]_1^5 \quad (=\ln 5)$	A1	For $\ln x$
	$\int x y\,\mathrm{d}x = \int_1^5 x\left(\frac{1}{x}\right)\mathrm{d}x \quad (=\left[x\right]_1^5 = 4)$	M1	*Limits not required*
	$\bar{x} = \frac{4}{\ln 5}$ *(2.485)*	A1	
	$\int \frac{1}{2}y^2\,\mathrm{d}x = \int_1^5 \frac{1}{2}\left(\frac{1}{x}\right)^2 \mathrm{d}x$	M1	For $\int\left(\frac{1}{x}\right)^2 \mathrm{d}x$
	$= \left[-\frac{1}{2x}\right]_1^5 \quad (=\frac{2}{5})$	A1	For $-\frac{1}{2x}$
	$\bar{y} = \frac{2/5}{\ln 5} = \frac{2}{5\ln 5}$ *(0.2485)*	A1 **7**	
(iii)	CM of R_2 is $\left(\frac{2}{5\ln 5}, \frac{4}{\ln 5}\right)$	B1B1 ft **2**	*Do not penalise inexact answers in this part*
(iv)		B1	For CM of R_3 is $(\frac{1}{2}, \frac{1}{2})$ (one coordinate is sufficient)
		M1	Using $\sum mx$ with three terms
	$\bar{x} = \frac{(\ln 5)\left(\frac{4}{\ln 5}\right) + (\ln 5)\left(\frac{2}{5\ln 5}\right) + (1)\left(\frac{1}{2}\right)}{\ln 5 + \ln 5 + 1}$	M1	Using $\frac{\sum mx}{\sum m}$ with at least two terms in each sum
	CM is $\left(\frac{4.9}{2\ln 5+1}, \frac{4.9}{2\ln 5+1}\right)$ *(1.161 , 1.161)*	A1 cao **4**	

4 (i)	$v = \frac{\mathrm{d}x}{\mathrm{d}t} = A\omega\cos\omega t - B\omega\sin\omega t$ $a = \frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$ $= -\omega^2(A\sin\omega t + B\cos\omega t) = -\omega^2 x$	B1 M1 E1 **3**	 Finding the second derivative Correctly shown
(ii)	$B = -16$ $\omega = 0.25$ $A = 30$	B1 B1 B2 **4**	When A is wrong, give B1 for a correct equation involving A [e.g. $A\omega = 7.5$ or $7.5^2 = \omega^2(A^2 + B^2 - 16^2)$] or for $A = -30$
(iii)	Maximum displacement is $(\pm)\sqrt{A^2 + B^2}$ Maximum displacement is 34 m Maximum speed is $(\pm)\ 34\omega$ Maximum acceleration is $(\pm)\ 34\omega^2$ Maximum speed is $8.5\ \mathrm{m\,s^{-1}}$ Maximum acceleration is $2.125\ \mathrm{m\,s^{-2}}$	M1 A1 M1 F1 F1 **5**	Or $7.5^2 = \omega^2(\mathrm{amp}^2 - 16^2)$ Or finding t when $v = 0$ and substituting to find x For either (any valid method) Only ft from $\omega \times$ amp Only ft from $\omega^2 \times$ amp
(iv)	$v = 7.5\cos 0.25t + 4\sin 0.25t$ When $t = 15$, $v = 7.5\cos 3.75 + 4\sin 3.75$ $= -8.44$ Speed is $8.44\ \mathrm{m\,s^{-1}}$ (3 sf); downwards	M1 A1 **2**	
(v)	Period $\frac{2\pi}{\omega} \approx 25$ s, so $t = 0$ to $t = 15$ is less than one period When $t = 15$, $x = 30\sin 3.75 - 16\cos 3.75$ $= -4.02$ Distance travelled is $16 + 34 + 34 + 4.02$ Distance travelled is 88.0 m (3 sf)	M1 M1 M1 A1 cao **4**	Take account of change of direction Fully correct strategy for distance