Jonny Griffiths

Paston College

Norfolk

Rich Tasks
in the Mathematics Classroom

How can open-ended and investigatory mathematics activities be used to deepen understanding and motivation for our students?

Know your audience!

Sixth Form College Teachers,

Secondary Teachers,

And?

Gatsby Teacher Fellow 2005-6

GTEP GATSBY TEACHER FELLOWSHIPS

The Gatsby Charitable Foundation

Risps – Rich Starting Points for A Level Mathematics

Open-ended investigatory activities that encourage exploration and a problem-solving approach

www.risps.co.uk

RISPs

The Website

What counts as a rich task?

Key element;

Some aspect of proof and play

Understanding distinctions

Routine/non-routine argument

Stimulates discussion

Risp 8. Arithmetic Simultaneous Equations

Pick six consecutive terms from an arithmetic sequence, and place them in order into the squares below.

□x + □y = □

□x + □y = □

Now solve this pair of simultaneous equations. What do you discover?

$x + 2y = 3$
$4x + 5y = 6$

$x = -1, y = 2$

$-2x - y = 0$
$x + 2y = 3$

$x = -1, y = 2$

Conjecture;

if the six coefficients are consecutive terms from an arithmetic progression, then the solution will be

$x = -1, y = 2.$

$ax + (a + d)y = a + 2d$
$(a + 3d)x + (a + 4d)y = a + 5d$

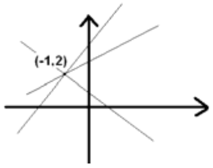
$x = -1, y = 2$

Converse;
if the solution to a linear simultaneous equation in x and y is x = -1, y = 2, then the six coefficients are consecutive terms from an arithmetic progression.
True?

$$4x + 5y = 6$$
$$8x + 7y = 6$$
$$x = -1, y = 2$$

Counter-example

The point (-1,2) is always on
$ax + (a+d)y = a+2d$



If (-1,2) is on $ax + by = c$,
then a, b and c are in arithmetic progression

Mini-theorem; the solution to a linear simultaneous equation in x and y is x = -1, y = 2 iff the top three coefficients are consecutive terms from an arithmetic progression, as are the bottom three.

Fibonnacci?

Geometric?

Three variables?

Use of computing power to reduce drudgery

Examples are self-generated
There is an element of ‘magic’ here
A rich task involves generalising, conjecture and proving a mini-theorem
Good mathematical processes are leading somewhere; there is a goal
Extensions suggest themselves
Easy teacher checking
Range of abilities in your classroom?
A rich task differentiates

If you divide the group into those

you see as likely to get A-B grades,

those likely to get C-D grades

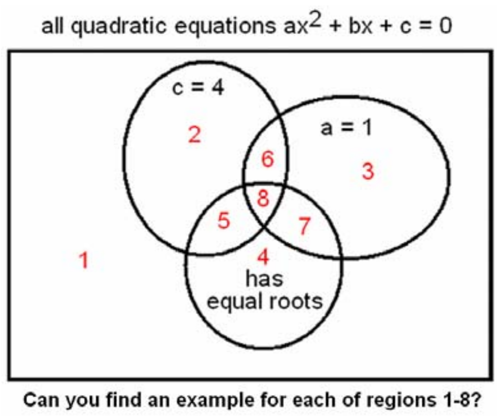
and those likely to get E-U grades,

a rough rule of thumb might be this:

The first third: immediately accessible
to A-B, C-D and E-U

The second third: immediately accessible
to A-B and C-D,
and moderately accessible to E-U.

The third third: immediately accessible
to A-B,
and moderately accessible to C-D.



Non-threatening

"Start with any quadratic at all!"

Puzzle element – "collect the set!"

Easy for the teacher

Differentiated

No guarantee everything is possible

Encourages discussion

Extendable

Understand distinctions

Overall description:
factors of 60
Property 1: multiples of 3
Property 2: multiples of 5
Property 3: multiples of 6

Overall description:
all lines y = mx + c
Property 1:
Line goes through (3, 0)
Property 2:
Line goes through (2, 4)
Property 3:
Lines are perpendicular

Overall description:
all binary operations on the set of real numbers excluding 0

Property 1: commutative
Property 2: associative
Property 3: closed

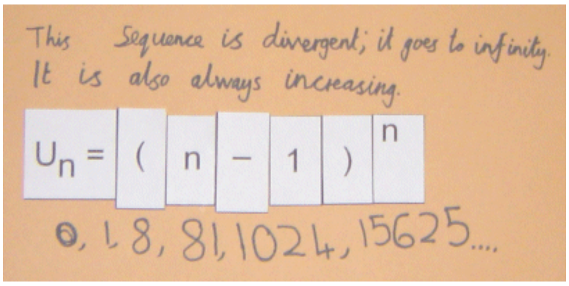
Arrange some or all of these tiles to define a sequence in as many different ways as you can.

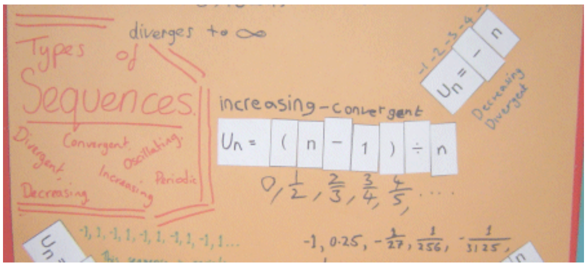
U_n =	n	)	n
1	(	÷	−

What different behaviours do these sequences exhibit?

Un = 1, 1, 1, 1, 1…
Un = -n, -1, -2, -3, -4…
Un = 1/n , 1, 1/2, 1/3, 1/4…
Un = n^n , 1, 4, 27, 256…
Un = $(-1)^n$, -1, 1, -1, 1…
Un = $(-n)^n$, -1, 4, -27, 256…
Un = $(-1)^n$ /n, -1, 1/2, -1/3, ¼…
Un = (n-1)/n , 0, 1/2, 2/3, ¾…

	Divergent	Convergent	Periodic	Other
Always increasing	n^n	(n-1)/n	NA	NA
Always decreasing	-n	1/n	NA	NA
Oscillating	$(-n)^n$	$(-1)^n/n$	$(-1)^n$	?
"Flat"	NA	$u_n = u_{n-1}$	NA	NA





Type the equation $y = ax^2 + bx + c$ into your graphing package.

Now mix up a, b and c to give six parabolas, one for each order.

Now try varying a, b, and c using the Constant Controller.

Do these six parabolas have anything in common?

Can you find a, b and c so that:

1. all the lines of symmetry are to the left of the y-axis?

2. all the lines of symmetry are to the right of the y-axis?

Need
$-a/2b$
$-b/2c$
$-c/2a$
to all be positive…

3. all six parabolas have two solutions for y = 0?

Need
$b^2>4ac$
$c^2>4ba$
$a^2>4bc$
Choose a –ve, b –ve
c large and +ve

Student response

Lesson 1

Theory then Practice

OR…

Lesson 2

Explore then Theory then Practice

90-minute lesson

Theory then Practice
45 mins theory, 45 mins practice

Explore then Theory then Practice
30 mins exploration
30 mins theory
30 mins practice

Lesson 1

Theory then Practice

Bread and butter

Lesson 2

Explore then Theory then Practice

Jam Sandwich

Bread, butter and jam.

- Prefer Lesson 2 – exploration is helpful as you begin to form conjectures by yourself and understand the subject better.
- Lesson 2 is better as it lets us visualise the theories and see how they are applied, even if we don't quite grasp what's going on before we are given the theory.
- Lesson 2! Exploration helped us to have a visual aid – made remembering better, and made you think about it. Theory and practice helped to cement it.
- The Explore part was good because it gives us a more practical approach and breaks up the lesson.
- Lesson 2 was helpful - and kind of fun!

'Prefer Lesson 1' responses were rarer;

But included this:

Lesson Two was scary.
We wanted to be told what to do.

Risps will solve all your problems!

Your students will love your lessons,

And they will all get brilliant grades…

maybe…

Potential Risp Problems

Unexpected outcomes can throw the teacher (and can confuse the student)

Is student curiosity fully present at the start of the activity?

The risp turns out to be more difficult than expected…

The teacher feels insecure

Much more skill and energy is required from the risps teacher

---

A good
Theory-Practice
lesson
is better than
a bad
Explore-Theory-Practice
lesson

Educationalists want us to teach in open-ended ways, to really make our students think hard mathematically…

So why are our exam questions so closed?

Risps do take time…

So why am I asked to teach a 60-hour module in 40 hours?

Risps and Exam Results

My results for 2005-6

More able students; fine

Less able students; worse

Risps are risky!

But…

Risps are what real mathematicians do

Conjecturing, Generalising,
Modelling, Discussing,
Making mistakes,
Problem-solving,
Refining, Proving, Waiting…

Risps are what
students remember

What was
the first investigation
you took part in?

Where do risps go right?

When consolidation is playful and active
and involves discussion

When the theory is constructed together
in response to a felt student need

When I set a task that is so engrossing that
my students forget I am in the room

So we should never
lose faith
when it comes to risps…

Why did I become a maths
teacher in the first place?