

**FREE-STANDING MATHEMATICS QUALIFICATION
ADVANCED LEVEL**

Additional Mathematics

6993

QUESTION PAPER

Candidates answer on the printed answer book.
OCR supplied materials:
• Printed answer book 6993
Other materials required:
• Scientific or graphical calculator

**Monday 13 June 2011
Morning**

Duration: 2 hours

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the printed answer book and the question paper.

- The question paper will be found in the centre of the printed answer book.
- Write your name, centre number and candidate number in the spaces provided on the printed answer book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the printed answer book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given correct to three significant figures where appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the printed answer book and the question paper.

- The number of marks is given in brackets [] at the end of each question or part question on the question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **100**.
- The printed answer book consists of **20** pages. The question paper consists of **8** pages. Any blank pages are indicated.

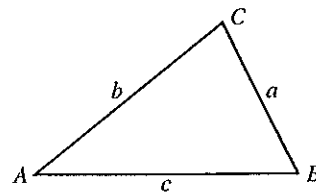
INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this question paper for marking; it should be retained in the centre or destroyed.

Formulae Sheet: 6993 Additional Mathematics

In any triangle ABC

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$



Binomial expansion

When n is a positive integer

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n$$

where

$$\binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

Answer all questions on the Printed Answer Book provided.

Section A

- 1 Determine whether the point (5, 2) lies inside or outside the circle whose equation is $x^2 + y^2 = 30$.
You must show your working. [3]
- 2 The equation of a curve is $y = x^3 - x^2 - 2x - 3$.
Find the equation of the tangent to this curve at the point (3, 9). [5]
- 3 In the triangle PQR, $PQ = 8$ cm, $RQ = 9$ cm and $RP = 7$ cm.
(i) Find the size of the largest angle. [4]
(ii) Calculate the area of the triangle. [3]
- 4 Solve the equation $5 \sin 2x = 2 \cos 2x$ in the interval $0^\circ \leq x \leq 360^\circ$.
Give your answers correct to 1 decimal place. [5]
- 5 The coordinates of the points A, B and C are (-2, 1), (5, 2) and (4, 9) respectively.
(a) Find the coordinates of the midpoint, M, of the line AC. [1]
(b) Show that BM is perpendicular to AC. [3]
(c) (i) Use the result of part (b) to state the mathematical name of the triangle ABC. [1]
(ii) Prove this by another method. [2]
- 6 Solve the inequality $x^2 - 12x + 35 \leq 0$. [4]
- 7 (a) Determine whether or not each of the following is a factor of the expression $x^3 - 7x + 6$.
You must show your working.
(i) $(x - 2)$ [2]
(ii) $(x + 1)$ [1]
(b) (i) Factorise the function $f(x) = x^3 - 7x + 6$. [3]
(ii) Solve the equation $f(x) = 0$. [1]

- 8 (i) On the axes given, indicate the region for which the following inequalities hold. You should shade the region which is **not** required.

$$5x + 3y \geq 30$$

$$3x + y \geq 12$$

$$y \geq 0$$

$$x \geq 0$$

[5]

- (ii) Find the minimum value of $6x + y$ subject to these conditions.

[2]

- 9 The gradient function of a curve is given by $\frac{dy}{dx} = 3x^2 - 2x + 4$.

Find the equation of the curve, given that it passes through the point (2, 2).

[4]

- 10 You are given that $\sin \theta = \frac{2}{5}$ with $0^\circ \leq \theta \leq 90^\circ$.

Using the identity $\sin^2 \theta + \cos^2 \theta = 1$, find an exact value for $\cos \theta$.

[3]

Section B

- 11 Eggs are delivered to a supermarket in boxes of 6.
For each egg, the probability that it is cracked is 0.05 independently of other eggs.

Find the probability that

- (i) in one box there are no cracked eggs, [2]
(ii) in one box there is exactly 1 cracked egg. [4]

The manager checks the eggs as follows.

- He takes a box at random from the delivery.
- He accepts the whole delivery if this box contains no cracked eggs.
- He rejects the whole delivery if the box contains 2 or more cracked eggs.
- If the box contains 1 cracked egg then he chooses another box at random.
- He accepts the delivery only if this second box contains no cracked eggs.

- (iii) Find the probability that the delivery is rejected. [6]

- 12 Two cars, A and B, move from rest away from a point O on a straight road starting at the same time.

- (a) Car A moves with constant acceleration of 2 m s^{-2} .

Express the displacement of car A after time t seconds as a function of t . [2]

- (b) Car B moves with acceleration given by $a = \frac{1}{2}t + 1$.

Express the displacement of car B after time t seconds as a function of t . [4]

- (c) (i) Find the time at which the cars are the same distance from O. [2]

(ii) Find the distance they have travelled at that time. [2]

- (d) Draw a sketch graph of the velocity of each car on the axes given. [2]

- 13 A pyramid has a square base, ABCD, with vertex E. E is directly above the centre of the base, O, as shown in Fig. 13.
 The lengths of the sides of the base are each $2x$ metres and the height is h metres.
 The lengths of the sloping edges, AE, BE, CE and DE, are each 5 metres.

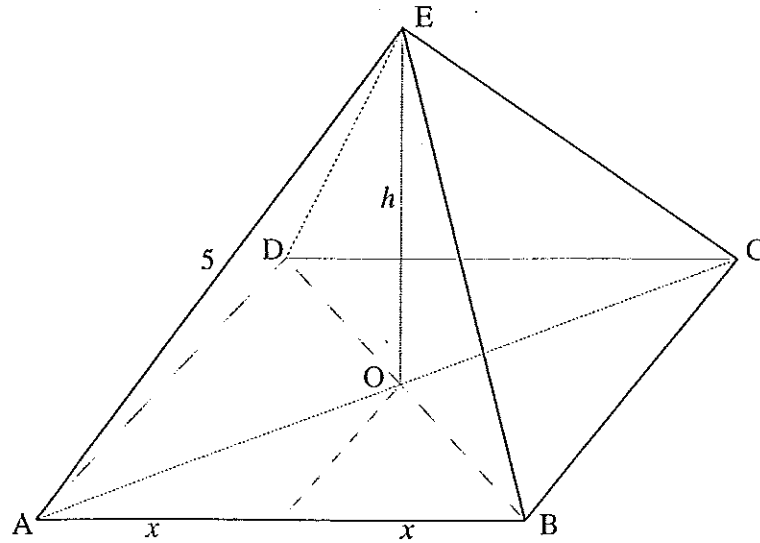


Fig. 13

- (i) Show that $2x^2 = 25 - h^2$. [2]
- (ii) Show that the volume of the pyramid, $V \text{ m}^3$, is given by $V = \frac{50h - 2h^3}{3}$. [2]
- (iii) As h varies, find the value of h for which V has a stationary value. [4]
- (iv) Prove that this stationary value is a maximum. [2]
- (v) Calculate the angle between the edge AE and the base when h takes this value. [2]

[Volume of a pyramid = $\frac{1}{3} \times \text{base area} \times \text{height}$.]

- 14 The cross-section of a speed hump is modelled by the region enclosed by the x -axis and the curve

$$y = \frac{1 - (x - 1)^4}{5}.$$

The graph is shown in Fig. 14.
Units are metres.

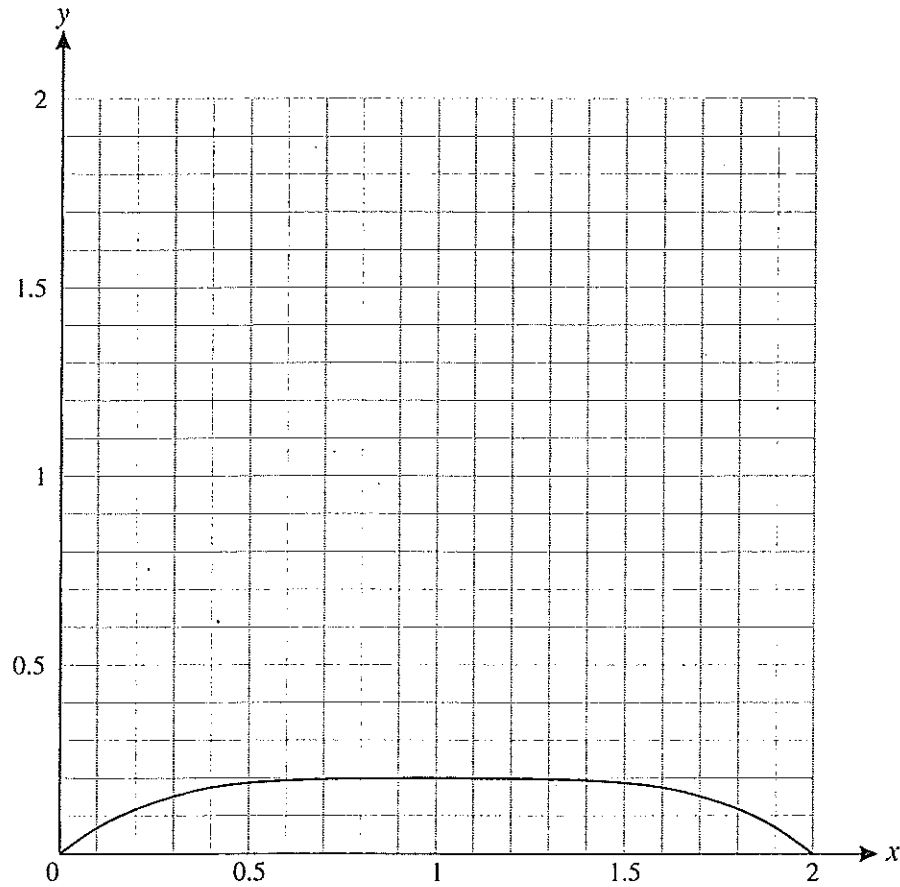


Fig. 14

- (a) (i) Write down the maximum value of $1 - (x - 1)^4$. [1]
 (ii) Hence write down the maximum height of the speed hump. [1]
- (b) Show that $y = \frac{1}{5}(4x - 6x^2 + 4x^3 - x^4)$. [3]
- (c) Find the area of the cross-section of the speed hump. [7]

Additional FSMQ

Free Standing Mathematics Qualification

6993: Additional Mathematics

Mark Scheme for June 2011

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by Examiners. It does not indicate the details of the discussions which took place at an Examiners' meeting before marking commenced.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the Report on the Examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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1. Subject-specific Marking Instructions

1. **M** (method) marks are not lost for purely numerical errors.
A (accuracy) marks depend on preceding **M** (method) marks. Therefore **M0 A1** cannot be awarded.
B (independent) marks are independent of **M** (method) marks and are awarded for a correct final answer or a correct intermediate stage.
2. Subject to 1, two situations may be indicated on the mark scheme conditioning the award of **A** marks or **B** marks:
 - i. Correct answer correctly obtained (no symbol)
 - ii. Follows correctly from a previous answer whether correct or not (**FT** on mark scheme and on the annotations tool).
3. Always mark the greatest number of significant figures seen, even if this is then rounded or truncated in the answer.
4. Where there is clear evidence of a misread, a penalty of 1 mark is generally appropriate. This may be achieved by awarding **M** marks but not an **A** mark, or awarding one mark less than the maximum.
5. For methods not provided for in the mark scheme give as far as possible equivalent marks for equivalent work. If in doubt, consult your team leader.
6. Where a follow through (**FT**) mark is indicated on the mark scheme for a particular part question, you must ensure that you refer back to the answer of the previous part question if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

2. Abbreviations

The following abbreviations are commonly found in Mathematics mark schemes.

- Where you see **oe** in the mark scheme it means **or equivalent**.
- Where you see **cao** in the mark scheme it means **correct answer only**.
- Where you see **soi** in the mark scheme it means **seen or implied**.
- Where you see **www** in the mark scheme it means **without wrong working**.
- Where you see **rot** in the mark scheme it means **rounded or truncated**.

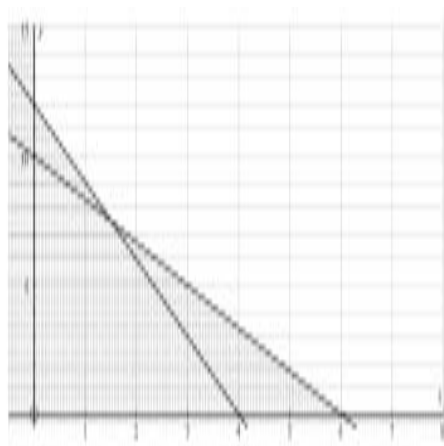
- Where you see **seen** in the mark scheme it means that you should award the mark if that number/expression is seen anywhere in the answer space, even if it is not in the method leading to the final answer.
- Where you see **figs 237**, for example, this means any answer with only these digits. You should ignore leading or trailing zeros and any decimal point e.g. 237000, 2·37, 2·370, 0·00237 would be acceptable but 23070 or 2374 would not.

Section A

Question		Answer	Marks	Part Marks and Guidance	
1		For (5, 2) use $x^2 + y^2 = 29$ so inside	M1 A1 A1 3	Substitute or use Pythagoras soi or $\sqrt{29}$ Conclusion (dependent on M1A1 awarded)	As usual only award A marks if the M mark has been awarded. Alternative method: Sub of $x = 5$ or $y = 2$ in $x^2 + y^2 = 30$ to find y or x M1 $y = \sqrt{5}$ or $x = \sqrt{26}$ A1
2		$\frac{dy}{dx} = 3x^2 - 2x - 2$ At $x = 3$ gradient = $27 - 6 - 2 = 19$ $\Rightarrow y - 9 = \text{"their"} 19(x - 3)$ $\Rightarrow y = 19x - 48$ oe	M1 A1 A1 M1 A1 5	Differentiate All three terms correct 19 isw (dep on first M1) Find line using correct point and <i>their</i> 19	At least one power decreased by 1. <i>"their"</i> 19 means: the value of the derivative Only 3 terms
3	i	eg $\cos P = \frac{8^2 + 7^2 - 9^2}{2 \cdot 8 \cdot 7}$ oe $\Rightarrow P = 73.4^\circ$	M1 A1 M1 A1 4	Cosine formula correctly used to find any angle Anything that rounds to 73.4° , 48.2° or 58.4° For identifying correct angle	Anything that rounds to 73.4°

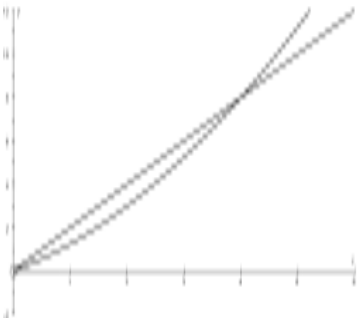
Question			Answer	Marks	Part Marks and Guidance	
3	ii		$\text{Area} = \frac{1}{2} \times 7 \times 8 \times \sin(\text{their angle } P)$ $= 26.8$	M1 A1 A1 3	Use of formula Correct substitution from <i>their (i)</i>	Can be at any vertex Anything that rounds to 26.8 Accept complete alternative methods
4			$5 \sin 2x = 2 \cos 2x$ $\Rightarrow \tan 2x = 0.4$ $\Rightarrow 2x = 21.8, 201.8$ $\Rightarrow x = 10.9, 100.9$ Also $x = 190.9, 280.9$	M1 A1 A1 A1 A1 5	Use of tan 0.4 10.9 Any 2 nd value 3 rd and 4 th values (<i>ignore extra solutions</i>)	allow $\tan x = 0.4$ for M1A1 Alternative method Use of Pythagoras to get $\sin 2x = \frac{2}{\sqrt{29}}$ or $\cos 2x = \frac{5}{\sqrt{29}}$ M1A1 and the last three marks are still available, ignore extra solutions
5	a		$M \text{ is } \left(\frac{-2+4}{2}, \frac{1+9}{2} \right) \text{ which is } (1,5)$	B1 1		
5	b		Gradient of AC is $\frac{9-1}{4+2} = \frac{4}{3}$ Gradient of BM is $\frac{2-5}{5-1} = -\frac{3}{4}$ $\frac{4}{3} \times -\frac{3}{4} = -1$ oe	B1 B1 B1 3	One gradient Second gradient <i>Their</i> $m_1 \times m_2 = -1$	Eg One is the negative reciprocal of the other
5	c	i	Isosceles	B1 1	Allow right-angled isosceles	Accept wrong spelling Do not accept right-angled triangle

Question			Answer	Marks	Part Marks and Guidance	
5	c	ii	$AB^2 = 7^2 + 1^2 = 50$ $BC^2 = 7^2 + 1^2 = 50$ \Rightarrow two sides equal in length	M1 A1 2	Using Pythagoras on AB and BC Or fully labelled diagram with correct sides shown	Attempt by vectors AB and BC M1 Alternative If answer to (c)(i) was right-angled, then accept proof that it is (requires all three lengths.) Alternative: If (c)(i) was equilateral or scalene then M1 (only) for attempt at all three sides. NB If nothing is written in (i) then no credit in this part.
6			$(x \pm 5)(x \pm 7)$ Boundaries $x = 5, x = 7$ $\Rightarrow 5 \leq x \leq 7$	M1 A1 B2 4	Or use of correct formula (allow one error in substitution) or correct shaped graph seen soi Accept $x \geq 5, x \leq 7$ for B1, B1	Condone < or >
7	a	i	Attempt to find $f(2)$ by substitution of 2 $= 0$, So Yes	M1 A1 2	Remainder theorem or attempt to divide (justification is sight of $x^3 - 2x^2$) Or: attempt to factorise, justification is sight of $(x^2 \dots 3)$ Correct working only	
7	a	ii	$f(-1) = -1 + 7 + 6 = 12$ so no.	B1 1	Sight of 12 or correct evidence, conclusion required	
7	b	i	$f(x) = (x - 2)(x^2 + 2x - 3)$ $= (x - 2)(x + 3)(x - 1)$	M1 A1 A1 3	Attempt to factorise or use long division (justifications as in (a)(i)) Sight of correct quadratic soi Answer	Alternative: Use of Remainder theorem M1 Sight of 2 nd factor A1 All correct A1
7	b	ii	$x = 1, 2, -3$	B1 1	FT their brackets	Must be three roots

Question		Answer	Marks	Part Marks and Guidance	
8	i		B1 B1 B1 B1 B1	for one line for correct shading for other line for correct shading for correct shading to give $x \geq 0, y \geq 0$	<i>If there is work here that is not crossed out, then mark it and ignore anything on Page 18.</i> Helpful hint: Lines go through (0, 12) and (4, 0) (0, 10) and (6, 0) Intersection at (1.5, 7.5)
8	ii	$6x + y$ is minimum at (0, 12) (<i>can be implied by correct answer</i>) So is 12	B1 B1 2		
9		$\frac{dy}{dx} = 3x^2 - 2x + 4 \Rightarrow (y) = x^3 - x^2 + 4x + c$ (2, 2) satisfies $\Rightarrow 2 = 8 - 4 + 8 + c$ $\Rightarrow c = -10$ $\Rightarrow y = x^3 - x^2 + 4x - 10$	M1 A1 M1 A1 4	Integrate Ignore c (dep on 1 st M1 mark) Substitute cao	At least one term with power increased by 1. (NB do not accept multiplying throughout by x) (ie must be $y = \dots$)
10		$\sin \theta = \frac{2}{5} \Rightarrow \sin^2 \theta = \frac{4}{25} \Rightarrow 0.16 + \cos^2 \theta = 1$ $\cos^2 \theta = \frac{21}{25}$ $\Rightarrow \cos \theta = \frac{1}{5} \sqrt{21}$ oe	M1 A1 A1 3	Use of Pythagoras $\cos^2 \theta$ eg $\sqrt{0.84}$ or $\sqrt{\frac{21}{25}}$ isw	Sight of a triangle with sides 2, 5, $\sqrt{21}$ acceptable for M1 Then A2 for $\cos \theta$ NB M0 if calculator used to find θ in order to find $\cos \theta$

Section B

Question		Answer	Marks	Part Marks and Guidance	
11	i	$P(0) = (0.95)^6$ $= 0.735(09189\dots)$	M1 A1 2	Correct p plus correct power	Not 2sf
11	ii	$P(1) = 6 \times (0.95)^5 \times (0.05)^1$ $= 0.232(134281\dots)$	M1 B1 B1 A1 4	Correct p and q and powers add to 6 Coefficient soi Correct powers for correct p and q soi	Coefficient may be missing
11	iii	$P(1^{\text{st}} \text{ box contains 2 or more eggs})$ $= 1 - (\text{their (i)} + \text{their (ii)})$ $= 1 - (0.7351 + 0.2321) = 1 - 0.9672 = 0.0328$ $P(2^{\text{nd}} \text{ box has any cracked eggs})$ $= 1 - \text{their (i)}$ $= 0.2649$ $P(\text{ consignment is rejected})$ $= 0.0328 + 0.2649 \times \text{their (ii)}$ $= 0.0328 + 0.0615$ $= 0.0943$	M1 A1 M1 A1 M1 A1 6	Accept anything rounding to 0.033 Accept anything rounding to 0.265 In either method, accept answers which lie between 0.094 and 0.095	Alternative $P(\text{accepted})$ M1 $\text{Ans(ii)} \times \text{Ans(i)}$ A1 0.1706 soi (Accept 0.171) M1(dep) Add to this Ans(i) A1 0.9057 (Accept 0.906) M1 $P(\text{consignment is rejected})$ $= 1 - 0.9057$ A1 $= 0.09428$

Question			Answer	Marks	Part Marks and Guidance	
12	a		$s = ut + \frac{1}{2}at^2$ with $u = 0$ and $a = 2$ $\Rightarrow s = t^2$	M1 A1 2	Constant acceleration formulae or integrate twice – ignore c	
12	b		$(v) = \frac{t^2}{4} + t$ $s = \frac{t^3}{12} + \frac{t^2}{2}$ Ignore c	M1 A1 M1 A1 4	Integrate Integrate	
12	c	i	$\frac{t^3}{12} + \frac{t^2}{2} = t^2$ $\Rightarrow \frac{t}{12} + \frac{1}{2} = 1$ $\Rightarrow t = 6$	M1 A1 2	Equate their functions	
12	c	ii	$s = 6^2$ or $s = \frac{6^3}{12} + \frac{6^2}{2}$ Displacement = 36 (m)	M1 A1 2	Substitute <i>their non-zero (c)(i)</i> in <i>their (a)</i> or <i>(b) soi</i>	
12	d			B1 B1 2	One clearly straight line through origin with positive gradient Other clearly a curve through the origin of correct shape with first part below the line as per diagram	Ignore labels

Question		Answer	Marks	Part Marks and Guidance	
13	i	$AO^2 = x^2 + x^2 = 2x^2$ or $AC^2 = (2x)^2 + (2x)^2 = 8x^2$ $h^2 + AO^2 = AE^2 \Rightarrow h^2 + 2x^2 = 25$ $\Rightarrow 2x^2 = 25 - h^2$	M1 A1 2	Correct application of Pythagoras on the base Algebra must be convincing	NB Answer is given
13	ii	$V = \frac{1}{3} \times \text{base area} \times \text{height} = \frac{1}{3} \times 4x^2h$ $= \frac{50h - 2h^3}{3}$	M1 A1 2	Formula seen including $4x^2$	Care: the answer is given
13	iii	$\frac{dV}{dh} = \frac{50 - 6h^2}{3}$ $= 0$ when $50 - 6h^2 = 0$ $\Rightarrow h^2 = \frac{25}{3}$ $\Rightarrow h = \sqrt{\frac{25}{3}} = \frac{5}{\sqrt{3}} = \frac{5}{3}\sqrt{3} = 2.89$	M1 A1 M1 A1 4	Differentiation cao dep Set (numerator) = 0 Any of these answers is acceptable	SC3 $h = 2.89$ with either $\frac{dV}{dh}$ or $\frac{1}{3}$ missing Numerical value must be 2.89
13	iv	$\frac{d^2V}{dh^2} = -4h$ < 0 so maximum	M1 A1 2	Or alternatives: Complete method to investigate value of derivative Or: complete method to investigate the value of V either side and at the turning point	Accept $-12h$
13	v	At this point $\sin EAO$ $\frac{h}{5} = \frac{1}{3}\sqrt{3}$ $\Rightarrow \text{Angle } EAO = 35.3^\circ$	M1 A1 2	Use of a correct ratio with <i>their</i> h (and/or x) Accept 35.2 which comes from $h = 2.88$	

Question			Answer	Marks	Part Marks and Guidance	
14	a	i	Max value = 1	B1 1		Not from any use of 0.2 from graph
14	a	ii	Height = 0.2 (m) or 20 cm	B1 1		
14	b		$x^4 - 4x^3 + 6x^2 - 4x + 1$ $\Rightarrow y = \frac{1}{5}(4x - 6x^2 + 4x^3 - x^4)$	B2 B1 3	-1 each error Dep on B2 convincing algebra (means sight of an extra correct step www)	An error is signs, powers, coefficients, failure to include the 1 at end
14	c		Area = $\int_0^2 \frac{1}{5}(4x - 6x^2 + 4x^3 - x^4).dx$ $= \frac{1}{5} \left[2x^2 - 2x^3 + x^4 - \frac{x^5}{5} \right]_0^2$ $= \frac{1}{5} \left(8 - 16 + 16 - \frac{32}{5} \right) = \frac{8}{25}$ $= 0.32$ Area of cross section = $0.32\text{m}^2 = 3200\text{cm}^2$	M1 A3 M1 A1 A1 7	Integrate (ignore c) A2 if one error, A1 if two errors (Dep on 1st M1) Deal with limits correctly (Putting $x = 0$ does not need to be seen) Units	Alternative method: Integrate original function is OK, but in dealing with limits $x = 0$ must then be seen. Omission of $\frac{1}{5}$ is one error. Multiply by $\frac{1}{5}x$ or $\frac{1}{5x}$, ie integrating $\frac{1}{5}$ gives A0

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