Evaluators' comments on 'Critical Maths': preliminary observations

Julian Williams, Manchester Institute of Education, The University of Manchester

Introduction

I was asked to act as an independent – but not necessarily disengaged or disinterested - evaluator and to make some early observations based on the papers circulated thus far (evaluation period June to Sept 2013).

Consequently I have worked with the following documents:

(i) papers selected by Terry Dawson including trial materials and reports of feedback from school pilots;
(ii) papers circulated to the Advisory Group;
(iii) I have also benefited from several meetings where the nature of the project and its progress was discussed.

It should also be mentioned that I have benefitted from engagement with a 'Curriculum and assessment for STEM' research study for the RS 'vision project' for STEM during this evaluation period (I can make this available to the AG if required, on a confidential basis). This work has informed some of the following remarks: particularly a propos "mathematical modelling", "lesson study R&D", and "student-led projects".

Of course these evaluations reflect my own dispositions and experience, which are broadly very sympathetic to the key notions in this project, namely: critical and creative thinking and problem solving; 'real' and valued applications and modelling; handling data; estimation and Fermi problems; widening participation and mathematics 'for all'; authentic assessment of higher level objectives; etc etc. I am pretty sure I share many of these principles with the members of the AG and the MEI team. Though I have concerns about the extent to which this project will be able to solve these age-old problems, I am entirely on the project's side in this ambition: 'be realistic, demand the impossible.'

Concerning Aims: motivation and emotion?

The project formulates aims for mathematics education that are related to good citizenship and should be core of all adolescents' education – indeed of education for all. There is a logic here for AS and A level but it is wise perhaps not to promote a discussion of this aspect at this stage.

The project also formulates higher order learning objectives that will be welcomed by many: these will pose worthy challenges, and not only for students. I would argue that it would help to give these higher order aims (which are currently somewhat disparate in the assessment objectives) some structure, in the form of a 'modelling process'.

There is also a vital focus on students' interest in problems posed (that have latent mathematical potential). While there is a need to ascertain empirically if the problems being developed by the project REALLY ARE motivating to the target students, and not just

intuitively expected to be motivating, there is also a need to develop the students own disposition to 'problem-pose' with mathematics. This is missing at the moment, probably because this is an early stage of development.

I think the aims relate to a worthy combination of 'emotions' and 'cognition' in the student experience: that is, they recognise the need for problems that 'move', i.e. appeal and motivate students to do/use mathematics, and hence to learn new mathematics. This could also be described with due reference to the social psychology of the developing adolescent: Vygotsky used the term "perezhivanie" (nearest English term might be 'emotional experience') as the unification of 'emotion' (the 'alpha and omega' in adolescence, according to Vygotsky) with the 'cognitive' in the learners' developmental experience.

Curriculum: and problem posing and modelling, but what could 'go'?

The curriculum and pedagogy being piloted is wisely focussed around particular problems and tasks so that a product that exemplifies the aims can be put in the hands of teachers to help them work with the target students. Without developing robust material the project cannot progress: one thinks of the effort to produce such material in the 1980s by the Shell Centre – eg the 'Blue box', which incidentally also produced video and specimen exam questions.

It is worth raising some warnings, however. There is a systemic tendency for such materials to limit pedagogy and become as structured and formulaic as the curriculum they aim to enrich: this is especially true if the assessment is too restrictive, as the test can constrain the pedagogy and learning outcomes (What You Test Is What You Get: more on this later).

Interestingly, the focus on 'an interesting problem that motivates some mathematics' can itself be limiting of the students' own creativity. One way to help the students creatively engage is to ensure that the problem is 'one of a class of problems' that can be dealt with by similar approaches; if the 'class of problem' is wide enough then the student can be expected to "pose a problem of your own to solve by this approach". In this way I would argue that one develops a 'mathematical disposition' - a mathematical way of looking at the world – something like the level 'does' in the Miller pyramid cited by the project.

For example: Fermi problems. The culmination of the lesson(s) involving Fermi estimation would best be "Here are a list of 10 problems/issues that could usefully be informed by a Fermi estimation... pose a Fermi problem of your own that you think interesting". I imagine this being the culmination of at least several weeks of work, possibly revisiting the material over the course of the year (or two?) of the course. It would be a useful exercise to try to see how this would work with each block of material in the course.

In addition, problems can been seen as starting points for a **mathematical modelling**[i] process of problem development and further model refinement: in this way sometimes students' creativity can be encouraged within constructive limits. I think there should always be plenty of room in these problem-solutions for critical evaluation of the modelling process, leading the student to reconsider the assumptions in the model and whether these need to be refined. I

think towards this end it will be helpful if some of the assessment criteria currently written under different sections were structured together – but more on assessment later.

Alas, I suspect that adding these dimensions to the curriculum may mean spending more time on tackling less material than at present (in what looks like a potentially busy curriculum for a fraction of an AS level) – but if necessary to reach the students' creativity and higher order challenge in terms of criticality then I would argue this is entirely beneficial. In fact, I would argue that the potential for a given 'class of problems' (with what we used to call a 'general mathematical competence' underpinning them) is one of the key criteria for establishing the significance for core content.

I suggest then that each unit or package is organised around a key model or mathematical competence that is associated with a 'range' or class of problems for which it is suitable, and that the unit is considered successfully taught if the students become aware both of the 'model' and the 'class of relevant problems and situations' in which they might use it. I would say this then leaves the learner with a metacognitive awareness as well as a general competence, and hopefully a disposition to see the world this way.

Thus, it might be argued that Fermi estimates and sampling principles are more important to the content than some others in that they are more generic and more likely to be engaged with in students' daily life and critical citizenship. The team might want to consider what is a priority and what material *could* go – I only offer some criteria here.

Prescribing pedagogy – a social model of the classroom?

It is not clear to me that it is necessarily wise or possible to prescribe a particular pedagogical approach to a programme that aspires to reach such a spread of population of learners, teachers, and institutions. The Socratic or Philosophy for Children approach is one that I very much favour (see Splitter & Sharp, 1995) but is open to mis- or multiple interpretations. One the one hand it can lead to a 'yes, Socrates –no, Socrates – three bags full, Socrates' that I'm sure the project does not have in mind. On the other, if focussed totally on the community of learners search for truth it can miss the targeted mathematics altogether. Having a prescribed but challenging and motivating task that 'begs to be organised' in Freudenthal's sense with a mathematics and supporting text seems to be your answer: good. The reference to neriage is also good: this implies some individual and small group work that is designed to lead to multiple suggestions that the teacher can use in whole group or whole class sharing. This can be very powerful and even exciting for the learners.

So what is the problem? There is a *but*, and it is a big but: the truth is that many teachers find neriage difficult, even with well-researched tasks in Japan. It is absolutely essential that the problem space is well mapped out and the likely student mathematical productions fed into supportive teacher guidance materials and training for teachers to anticipate responses to tasks. In Japan this has been the main focus of much lesson study research in mathematics in recent decades (more of this below, see also Hart, Alston, & Murata, 2011).

I fear this is particularly likely to be the case in post-16 educational contexts, where problem – solving has not been recently prominent, and where well-qualified teachers are in very short supply. After development, it may be that more teachers will gain confidence and skill in 'thinking on their feet', but this is a very significant development that cannot be expected from the start.

Assessment: over-ambitious or unambitious?

The project is already running to meet the critical issue of assessment before it has started to walk in the classroom: it was ever thus and I can't criticise the team for this, but there are dangers in this rush that the assessment will be unambitious, or too ambitious... or both in different places.

The desire to protect the classroom activity from summative assessment is entirely consistent with our research review (Howes et al., under review) finding about the power of **formative assessment** and the need for space-and-time for learners to engage with teaching and the task without being dominated by the discourse of the ever-approaching test/examination. However, teachers have become used in FE Colleges and 6th forms to discussing grade criteria in regard to new content on a lesson-by-lesson basis – this is how they perceive they meet the demands of performativity mentioned in your documents. Almost certainly this will remain the case in the new context with regard to exam tasks: only in the coursework tasks will there be learner-centred space: this is also the experience of our research in the pedagogy in the "Use of mathematics" AS (Hernandez-Martinez et al., 2011).

Even in the specimen examination materials being discussed however there may be some tricks being missed here. For instance there might be more questions that ask for critique of a given solution: these can often demand quite a bit of reading time (e.g. the Cameron question) but if necessary written material can be supplied in advance of the exam. Experience has shown this takes a certain amount of expertise not to encourage question-spotting, but it can be done.

Additionally, there might be open questions that ask for reflection on tasks the students might all be assumed to have done in advance in class, thus reaching higher order challenges or even the 'does' category in reflection. Taking the Isle of White population task as a basis, for instance, one might ask for a reflection on any ONE of the modelling assumptions that had proved controversial, or how they had been able to transfer the model/method used to another task.

Professional development: lesson study models of research and CPD?

Without wishing to dampen the enthusiasm of all with the realism of the decrepit author, the project will ossify at the end of its funding unless a self-sustainable curriculum development model is established through the project.

The plan to involve teachers and students through trialling materials and feeding back to the team is a good stimulus and will provide some essential grounding that can make for

reasonably robust materials. However, the project challenges to curriculum, teaching/pedagogy, and assessment almost certainly will require ongoing work and support.

I recommend that a formal system of lesson study be promoted where possible – these groups would research the pedagogy involved, potentially going beyond 'giving team feedback' to focus on their own concerns. There are a number of such groups being established (eg within the remit of NCETM) and work with some established groups might be helpful if this is possible.

Other aspects of the development process: Watford Grammar or Moss Side FEC?

I have not been able to gather the spread of 'target students' that will be involved in the pilot and development process and form whom the project will get feedback. I suspect that there is an inevitable risk of drawing on the willing and competent (perhaps even affluent?) schools, teachers, and students. I am not sure how to protect the project from such bias, but being aware of this possibility would tend to suggest always to make entry to the tasks as accessible as possible for teachers and students.

Designing an evaluation research, 2013-15[ii]

In thinking about evaluation methodologies, the project might wish to complement the use of some bespoke measuring instruments that capture the students' and teachers' 'mathematical disposition' alongside test and exam results: these could provide some multidimensional measurement data for informing policy (tapping disposition as well as cognitive outcomes is particularly crucial according to our research, see Pampaka et al., 2012).

In addition, however the evaluation could usefully feed into the teachers' CPD and lesson study through action-research type case studies that capture the complexity of pedagogy and assessment practice with the perezhivanie of the teachers and students. This might best happen through my (my team's) involvement with lesson study groups (if they should come into being). I suggest an online Action-Research practitioner journal of experiences/case studies that speak almost directly form teacher to teacher (akin to that proposed for the NCETM 'multiplicative reasoning' lesson study group).

Conclusion

The project has produced a challenging agenda and some exciting material over a brief period. I have tried to point to potential difficulties in the hope that anticipation will help avoid stumbling. In brief: Modelling; Problem posing; Formative assessment free from summative; Coursework; Lesson study research and CPD; Evaluation research.

Bibliography

Freudenthal, H. (1991) Revisiting mathematics education.Dordrecht, Academic pubs.

Hart,L., Alston, A. & Murata, A. (2011). Lesson study research and practice in mathematics education. Dordrecht: Springer.

Howes et al. (accepted for pub 2014) Re-envisioning STEM education. Royal Society report

Hernandez-Martinez, P., Williams, J., Black, L., Davis,P., Pampaka M., and Wake, G. (2011) "Mathematics coursework as facilitator of formative assessment, student-centred activity and understanding." *Research in Mathematics Education13*, (2), 197-212.

Pampaka, M., Williams,J. , Hutcheson,G., Wake,G., Black,L., Davis, P., Hernandez-Martinez., P. (2012) The association between mathematics pedagogy and learners' dispositions for university study." *British Educational Reseach Journal* ,38,3: 473-496.

Splitter, L. & Sharpe, A. (1995) Teaching for better thinking. Melbourne: ACER.

Notes

[i] On modelling: the suggestion is not that this should add any mathematical content, but just that the modelling cycle could provide a structure to the problem-solving processes the students can expect to pass through, and that this might situate the various processes such as “making estimates and introducing variables” at one end and “evaluating the solution, and reconsidering assumptions” at the other.

[ii] Further to a recent discussion with MEI team, I have written some notes that should help shape evaluation proposals for 2013-14, and 2014-15.