Duration: Up to 2 hours

Maximum Mark 72

Final Version
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This document consists of 17 pages
MARKING INSTRUCTIONS

PREPARATION FOR MARKING

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: scoris assessor Online Training; OCR Essential Guide to Marking.

2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal http://www.rm.com/support/ca

3. Log-in to scoris and mark 10 practice responses (“scripts”) and 10 standardisation responses

YOU MUST MARK 10 PRACTICE AND 10 STANDARDISATION RESPONSES BEFORE YOU CAN BE APPROVED TO MARK LIVE SCRIPTS.

TRADITIONAL

Before the Standardisation meeting you must mark at least 10 scripts from several centres. For this preliminary marking you should use pencil and follow the mark scheme. Bring these marked scripts to the meeting.

MARKING

1. Mark strictly to the mark scheme.

2. Marks awarded must relate directly to the marking criteria.

3. The schedule of dates is very important. It is essential that you meet the scoris 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

4. If you are in any doubt about applying the mark scheme, consult your Team Leader by telephone or the scoris messaging system, or by email.

5. Work crossed out:
   a. where a candidate crosses out an answer and provides an alternative response, the crossed out response is not marked and gains no marks
   b. if a candidate crosses out an answer to a whole question and makes no second attempt, and if the inclusion of the answer does not cause a rubric infringement, the assessor should attempt to mark the crossed out answer and award marks appropriately.
6. Always check the pages (and additional objects if present) at the end of the response in case any answers have been continued there. If the candidate has continued an answer there then add a tick to confirm that the work has been seen.

7. There is a NR (No Response) option. Award NR (No Response)
   - if there is nothing written at all in the answer space
   - OR if there is a comment which does not in anyway relate to the question (e.g. ‘can’t do’, ‘don’t know’)
   - OR if there is a mark (e.g. a dash, a question mark) which isn’t an attempt at the question

Note: Award 0 marks - for an attempt that earns no credit (including copying out the question)

8. The scoris comments box is used by your team leader to explain the marking of the practice responses. Please refer to these comments when checking your practice responses. **Do not use the comments box for any other reason.** If you have any questions or comments for your team leader, use the phone, the scoris messaging system, or e-mail.

9. Assistant Examiners will send a brief report on the performance of candidates to your Team Leader (Supervisor) by the end of the marking period. The Assistant Examiner’s Report Form (AERF) can be found on the RM Cambridge Assessment Support Portal (and for traditional marking it is in the Instructions for Examiners). Your report should contain notes on particular strength displayed as well as common errors or weaknesses. Constructive criticism of the question paper/mark scheme is also appreciated.

10. [DOES NOT APPLY TO GCE MATHEMATICS] For answers marked by levels of response:
    a. To determine the level – start at the highest level and work down until you reach the level that matches the answer
    b. To determine the mark within the level, consider the following:

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>Award mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>On the borderline of this level and the one below</td>
<td>At bottom of level</td>
</tr>
<tr>
<td>Just enough achievement on balance for this level</td>
<td>Above bottom and either below middle or at middle of level (depending on number of marks available)</td>
</tr>
<tr>
<td>Meets the criteria but with some slight inconsistency</td>
<td>Above middle and either below top of level or at middle of level (depending on number of marks available)</td>
</tr>
<tr>
<td>Consistently meets the criteria for this level</td>
<td>At top of level</td>
</tr>
</tbody>
</table>
11. **Annotations and abbreviations**

<table>
<thead>
<tr>
<th>Annotation in scoris</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ and ✗</td>
<td></td>
</tr>
<tr>
<td>BOD</td>
<td>Benefit of doubt</td>
</tr>
<tr>
<td>FT</td>
<td>Follow through</td>
</tr>
<tr>
<td>ISW</td>
<td>Ignore subsequent working</td>
</tr>
<tr>
<td>M0, M1</td>
<td>Method mark awarded 0, 1</td>
</tr>
<tr>
<td>A0, A1</td>
<td>Accuracy mark awarded 0, 1</td>
</tr>
<tr>
<td>B0, B1</td>
<td>Independent mark awarded 0, 1</td>
</tr>
<tr>
<td>SC</td>
<td>Special case</td>
</tr>
<tr>
<td>^</td>
<td>Omission sign</td>
</tr>
<tr>
<td>MR</td>
<td>Misread</td>
</tr>
</tbody>
</table>

| Highlighting         |         |

<table>
<thead>
<tr>
<th>Other abbreviations in mark scheme</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>Mark for explaining</td>
</tr>
<tr>
<td>U1</td>
<td>Mark for correct units</td>
</tr>
<tr>
<td>G1</td>
<td>Mark for a correct feature on a graph</td>
</tr>
<tr>
<td>M1 dep*</td>
<td>Method mark dependent on a previous mark, indicated by *</td>
</tr>
<tr>
<td>cao</td>
<td>Correct answer only</td>
</tr>
<tr>
<td>oe</td>
<td>Or equivalent</td>
</tr>
<tr>
<td>rot</td>
<td>Rounded or truncated</td>
</tr>
<tr>
<td>soi</td>
<td>Seen or implied</td>
</tr>
<tr>
<td>www</td>
<td>Without wrong working</td>
</tr>
</tbody>
</table>
12. **Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand**

a Annotations should be used whenever appropriate during your marking.

   **The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

   Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

   **M**
   A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

   **A**
   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

   **B**
   Mark for a correct result or statement independent of Method marks.

   **E**
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d  When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise: and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e  The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f  Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g  Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
1 (i) Periodic with period $2\pi$
Maximum 2, minimum 0
Comment about symmetry

B1 B1 B1 G2
Both required for mark. Accept comment about coordinates of maxima/minima.

Minima (zero gradient) clearly shown.
Give B1 for general shape correct (three cycles).

1 (ii) $\frac{dy}{dx} = \frac{\sin t}{1 - k \cos t}$
Denominator > 0 for all $t$.

OR

$\frac{dx}{dt} = 1 - k \cos t$
$\frac{dx}{dt} > 0$ for all $t$.

M1 A1 E1 E1
Evidence of attempt to find $\frac{dy}{dx}$.
Accept “denominator is never 0”.

M1 A1
Accept “is never 0”.
1 (iii) The curve has cusps.

G2 Periodic with three complete cycles. Cusps clearly shown.

B1

1 (iv) \[
\lim_{t \to 0^+} \left( \frac{\sin t}{1 - \cos t} \right) \to +\infty \\
\lim_{t \to 0^-} \left( \frac{\sin t}{1 - \cos t} \right) \to -\infty
\]

The point is defined at \( t = 0 \): \( x = 0, y = 0 \).

Therefore the curve has a cusp at (0,0)

M1 Limit from one direction only scores M1M0A0

A1

E1

E1

[5]
(v) \( \frac{dy}{dx} \) is infinite when \( 1 - 2 \cos t = 0 \)

\[ t = \frac{\pi}{3} \]

\[ x = \frac{\pi}{3} - 2 \sin \frac{\pi}{3} \]

\[ x = -\left( \sqrt{3} - \frac{\pi}{3} \right) \]

Hence width of loop is \( 2 \left( \sqrt{3} - \frac{\pi}{3} \right) \)

\[ = 2\sqrt{3} - \frac{2\pi}{3} \]

1 (vi) \[ \arccos \left( \frac{1}{k} \right) - k \sin \left( \arccos \left( \frac{1}{k} \right) \right) = -\pi \]

\[ k = 4.6033 \]

Any correct value of \( t \).

Substituting \( \frac{\pi}{3} \) to find width of half the loop.

Taking positive value and doubling.

For lhs correct but rhs = +\( \pi \) score M1A0A0

More solutions may exist. Try specifying appropriate...
\[ z = \frac{\sqrt{3} + 2}{2} - \frac{1}{2}i, \quad \frac{-(\sqrt{3} - 2)}{2} - \frac{1}{2}i \]

\[
\frac{\sqrt{3} + 2}{2} - \frac{1}{2}i \left( \frac{-(\sqrt{3} - 2)}{2} - \frac{1}{2}i \right) = \frac{\sqrt{3} + 2}{2} - \frac{1}{2}i(1 - 2i) = \\
\frac{-(\sqrt{3} - 2)}{2} - \frac{1}{2}i(1 - 2i) = \sqrt{3}
\]

B2  B1 for 1 or 2 correct only.

B2  Points marked correctly. B1 for two correct.

M1  Evaluation of the distance between the points.

A1  All shown to be equal.

Any other acceptable method should be awarded full marks.
2 (ii) \[ f'(z) = 3z^2 + (-6 + 6i)z - 6i \]

\[ f'(z) = 3(z - (1 - i))^2 \]. Repeated root at \( z = 1 - i \).

or

Discriminant \( =(-(6 + 6i))^2 - 4 \times 3 \times (-6i) \)

\[ = 0 \]

or

\[ \text{csolve}(3z^2 - (6 + 6i)z - 6i = 0, z) = 1 - i \]. This root must be repeated as a quadratic will have two roots over the complex numbers.

B1 M1 A1

Factorising \( f'(z) = 0 \).

Must show that the root is repeated for these marks by either factorising, finding the discriminant or stating that it must have two solutions over the complex numbers.

“\[ \text{csolve}(3z^2 - (6 + 6i)z - 6i = 0, z) = 1 - i \]” without further explanation scores M0A0.

A1

Point marked correctly.
### Question 2
#### (iii)
\[ g'(z) = 3z^2 + bz + c \]
Discriminant: \( b^2 - 12c \)
Discriminant = 0 \( \Rightarrow c = \frac{b^2}{3} \)

- **M1**
- **M1**
- **A1**

Accept \( b = \pm \sqrt{3c} \)
Award full marks for integrating \( k(\alpha - z)^2 \) and finding the same result by comparing coefficients.

#### (iv)
Function that has distinct roots, repeated root to \( f'(z) = 0 \).
Showing that the roots are in an equilateral triangle.

Function that has roots that are not in an equilateral triangle and distinct roots to \( f'(z) = 0 \).

- **B1**
- **B1**
- **B1**

[3]
2 (v) \[ |\beta| = 1 \text{ and } \arg(\beta) = \pm \frac{\pi}{3} \]
\[ \beta = \left( \frac{1}{2} \pm \frac{\sqrt{3}}{2} i \right) \text{ or } \beta = e^{\pm \frac{i\pi}{3}} \]
M1 Evidence of attempt to find modulus and argument or suitable diagram.
A1 A1 A1 [3]
Either form acceptable.

2 (vi) \[ h(z) = z(z - z_1) \left( z - \left( \frac{1}{2} + \frac{\sqrt{3}}{2} i \right) z_1 \right) \]
\[ h'(z) = 3z^2 - 3z_1z + \frac{z_1^2}{2} - \frac{z_1 \sqrt{3}(2z - z_1)}{2} i \]
\[ = \frac{(6z - z_1(3 + \sqrt{3}i))^2}{12} \]
So \( h'(z) = 0 \) has a single repeated solution.
M1 Or equivalent \( e^{i \frac{\pi}{3}} \) form
A1
Accept the alternative method of using the relationship between \( b \) and \( c \) from (iii) acceptable.
| 3 | (i) | Example program:  
Define program1(m)=  
  Prgm  
  Local i,j  
  For i,1,m  
  For j,1,m  
  If i^2-3*j^2=1 Then  
  Disp i,j  
  EndIf  
  EndFor  
  EndFor  
  EndPrgm  
  
  \(x = 2, y = 1; x = 7, y = 4; x = 26, y = 15; x = 97, y = 56\) | M6 | If some (or all) of the answers are incorrect allocate method marks as follows:  
M1 Appropriate structure program: nested loops or equivalent  
M1 Appropriate use of variables  
M1 Maximum value or 100 used.  
M1 Loop for \(x\) or equivalent  
M1 Loop for \(y\) or equivalent  
M1 Check (If) statement  
More efficient programs are possible.  

A4 | A1 for each correct answer.  
Subtract (maximum) A1 if additional solutions are given outside the range. | [10] |
| 3 (ii) | Using \( x = 2, y = 1 \).
\[
\begin{align*}
x_2 + y_2\sqrt{3} &= (2 + \sqrt{3})^2 \\
&= 7 + 4\sqrt{3}
\end{align*}
| M1 |
| A1 |
| Evidence of using \( x = 2, y = 1 \) to find at least 1 other solution. At least one shown correctly. |

So \( x = 7, y = 4 \) is a solution.

\[
\begin{align*}
x_3 + y_3\sqrt{3} &= (2 + \sqrt{3})^3 \\
&= 26 + 15\sqrt{3}
\end{align*}
| A1 |
| Other two shown to be solutions based on \( x = 2, y = 1 \). |

So \( x = 26, y = 15 \) is a solution,

\[
\begin{align*}
x_3 + y_3\sqrt{3} &= (2 + \sqrt{3})^4 \\
&= 97 + 56\sqrt{3}
\end{align*}
| A1 |
| Must be based on appropriate rearrangement. |

So \( x = 97, y = 56 \) is a solution,

If \( x^2 - 3y^2 = 1 \) then \( x^2 = 3y^2 \) \( \therefore \frac{x}{y} = \sqrt{3} \)

\[
\begin{array}{ccc}
2 & 7 & 26 \\
4 & 15 & 56 \\
\end{array}
| [5] |

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| **3 (iii)** | Change input to  
Define program1(m,n)=  
Change  
If \(i^2-3j^2=1\) Then to  
If \(i^2-nj^2=1\) Then  
Smallest solution is  \(x = 9, y = 4\)  
\(\frac{9}{4}\) is not accurate enough.  
Finding another solution (program or squares…)  
\(x = 161, y = 72\)  
\(\frac{161}{72}\) is accurate to within 0.1%  
P1       A1  
P1       Seen or implied  
|   |   | M1       Accept method based on changing to  
\(i^2-5j^2=1\) if clear.  
| **3 (iv)** | Writing \(n = d^2\) would give any solution as  
\(x^2 = (dy)^2 + 1\)  
But there are no square numbers that are 1 apart.  
|   |   | E2       [2]  
|   |   |   |