**Finding the Top Speed of a Train**

### About the company:

I work for Eversholt Rail – a train leasing company that owns about a third of all passenger trains in the UK. As owners of the rolling stock it is important to make sure they are highly competitive when trying to win leases with train operating companies. One way of doing this is to make the trains as fast as possible, meaning they can cover long distance journeys a lot quicker.

### The problem:

A train owner has a train with max speed 100mph (44.7m/s). They want to increase its maximum speed but are not sure how much faster the train can go. Finding a piece of flat track that isn’t busy where they can test its top speed is expensive. They would prefer to check by a cheaper method.

Above is a tractive effort curve typical of a new electric train. Tractive effort curves show how much force the wheels can push forward on the rails with for any given speed.

\[
\text{Force} = \text{Mass} \times \text{Acceleration} \quad \text{(From Newton’s second law)}
\]

Using the equation above you can work out the equivalent accelerations to each tractive effort. The vertical axis on the right shows you the equivalent acceleration assuming there is no resistance to motion. I have made this vehicle weigh 140 tonnes – typical of a 4-car train.

These equations do not prove to be accurate in reality as the resistance to motion acting on the train will act against its tractive effort.

For simulations and calculations a quadratic formula is used to represent resistance to motion. This is shown below.

\[
\text{Resistance to motion} (kN) = A + B \times \text{Speed} + C \times (\text{Speed})^2
\]

A, B and C are constants that change depending on the train. A is related to the friction acting between the rolling wheel and the rail, B is related to the friction produced by the internal moving parts of the motor and C is related mainly to air resistance. These values can range from train to train but for this one A=1.3, B=0.06 and C=0.006

This can then be plotted on the same graph as the tractive effort curve as shown over the page.
I have extended the speed range on the graph to show the point where the resistance to motion gets larger than the tractive effort. The speed at which both forces are equal is the maximum speed the train can get to on a flat piece of track. The maximum speed can roughly be calculated by reading the point at which tractive effort = resistance to motion on the graph.

A more accurate method of finding the maximum speed uses an additional equation that applies to the curved part of the tractive effort graph:

\[
\frac{Power}{Speed} = \text{Ttractive effort}
\]

The power of the train is 1450 kW. Using this equation and the resistance to motion equation we can form the equation below when tractive effort = resistance to motion:

\[
\frac{1450}{Speed} = A + B \times Speed + C \times (Speed)^2
\]

This can be rearranged to:

\[0.006 \times (Speed)^3 + 0.06 \times (Speed)^2 + 1.3 \times Speed - 1450 = 0\]

We can then solve this cubic through various methods to get Speed=58.03 m/s. That’s 208.9kph or 129.8mph!

**Question**

Because the acceleration at very high speeds gets very low, the top allowed speed is 5mph lower than the original limit because it would take too long to reach this speed. The owner would like the train to be able to be allowed to do 125mph. He is looking at making the train more aerodynamic to help increase its top speed. What is the highest value of C (air resistance) with which he can achieve this speed?

**Answer**

\[C \times (Speed)^3 + 0.06 \times (Speed)^2 + 1.3 \times Speed - 1450 = 0\]

\[Speed = 130mph = 58.1m/s\]

\[C \times (58.1)^3 + 0.06 \times (58.1)^2 + 1.3 \times 58.1 - 1450 = 0\]

So \(C = 0.00598\) (3s.f.)