GCSE Mathematics retake for vocational students

GCSE teaching from 2015

1. Introduction

MEI was commissioned by the DfE to look at the proposed content of the new GCSE Mathematics for teaching from September 2015 in relation to the needs of post-16 students who have not achieved GCSE Mathematics at grade C, or above. These students need to continue working towards GCSE Mathematics from August 2013 as part of study programmes.

This paper considers how the proposed new GCSE Mathematics might meet the needs of post-16 vocational education students who have not achieved a grade C by age 16. It explores how vocational contexts can be used in teaching GCSE Mathematics to vocational students, drawing on examples of good practice from the England and beyond.

2. Executive summary

2.1 Outline of content

Relevant recommendations from The Wolf Report, the RSA report, *Solving the Maths Problem*, an Ofsted survey of post-16 numeracy provision and the Sutton Trust report, *The Employment Equation*, are reviewed.

Consideration is given to some of the concerns about teaching GCSE Mathematics retake to post-16 vocational students and to how these might be addressed through professional development and appropriate resources.

Examples of good practice in contextualising mathematics for vocational students are given and an outline of different ways of using context in teaching mathematics is provided.

a. A realistic problem for students to solve, using skills they have already acquired.
b. A realistic problem for students to solve in order to motivate and facilitate the learning of new skills.
c. A realistic context to enable the students to see the point of the mathematics they are learning.
d. A realistic context to help students make sense of abstract mathematics.
e. A pseudo-context which looks as though it refers to real-life at first sight but does not.

The use of context in examination questions at this level is also outlined.

The methodology and results of a United States randomised controlled trial of using contexts to teach mathematics to vocational students suggests one way in which relevant contexts can be used to introduce mathematical content to vocational students, leading to their using the mathematics in more general contexts or without a context.

Vocational education, and the place of mathematics within it, is a matter of interest in many countries. The Nuffield Report, *Towards Universal Participation in post-16 Mathematics*, identified Germany and Singapore as having successful vocational education systems; the place of mathematics in these systems is described.

The varied and diverse nature of vocational education in England makes it difficult for end users to be familiar with all the vocational qualifications relevant to their field of work. In addition to being useful to vocational students by virtue of the learning it represents, GCSE Mathematics can provide them with an easily recognised and transferable qualification. It is important that vocational students and their advisers are aware of this.

Content that is less relevant for vocational students is identified and examples of real world contexts are given to support the teaching of most of the draft GCSE content

2.2 Conclusions and recommendations

- In addition to giving them important skills for life and work, GCSE Mathematics can provide vocational learners with a qualification which is widely recognised across all sectors of industry. It is important that students, families and teachers understand the transferability of GCSE Mathematics and the skills it represents so that they appreciate its value in vocational programmes.

- Students training for some vocations, e.g. engineering, will need mathematics beyond GCSE, especially if they want to progress in their chosen professions. Programmes of study for vocational students should include any mathematics beyond GCSE which they will need in order to progress in their careers.

- The use of vocational contexts in teaching GCSE Mathematics to retake students can enable them to re-engage with mathematics which they have previously found difficult. In addition to increasing their chances of GCSE success, this can also improve their ability to use appropriate mathematics at work.

- Introducing mathematical content to resit students through the use of contexts allows assessment for learning to take place through discussion of what students already know and so enables teaching to focus on what students need to know.

- Some GCSE Mathematics content lends itself to a wide variety of vocational applications; other content has fewer vocational applications. At the beginning of a vocational course, students should encounter contexts which are relevant to their chosen vocational study but they should also expect to work in a wider variety of contexts during the course.

- High quality teaching resources need to be developed for vocational students retaking GCSE Mathematics. Publishers will only develop resources which will sell well; recent funding changes in FE make this market more uncertain for them.

- Mathematics teachers and vocational teachers should be encouraged to work together to explore appropriate contexts for teaching mathematics to vocational students.

- Lecturers in FE need access to GCSE Mathematics CPD. Appropriate CPD is also needed for teachers in schools and sixth form colleges who will be teaching post-16 GCSE retake students, to help them adopt teaching strategies which will enable their students to succeed.

- CPD for teachers of GCSE retake students should enable them to teach post-16 GCSE Mathematics with confidence and introduce them to ways of using contexts in their teaching, in order to improve student engagement and understanding.

3. Background

3.1 The Wolf Report

The Wolf Report[1] highlighted the importance of GCSE Mathematics and English.

> *English and Maths GCSE (at grades A*-C) are fundamental to young people's employment and education prospects. Yet less than 50% of students have both at the end of Key Stage 4 (age 15/16); and at age 18 the figure is still below 50%. Only 4% of the cohort achieve this key credential during their 16-18 education. Worse, the funding and accountability systems established by government create perverse incentives to steer 16+ students into inferior alternative qualifications.*

The report made the following recommendation.

> *Students who are under 19 and do not have GCSE A*-C in English and/or Maths should be required, as part of their programme, to pursue a course which either leads directly to these qualifications, or which provides significant progress towards future GCSE entry and success.*

New study programmes are being introduced from September 2013 for students aged 16-19. It is a requirement of these study programmes that students who do not have GCSE Mathematics and English at grade A*-C should work towards obtaining them. In the case of Mathematics, FSMQs or Functional Skills qualifications can be used as "stepping stones" towards GCSE for students who are not yet ready to retake GCSE Mathematics[2].

3.2 The RSA Report, Solving the Maths Problem

GCSE Mathematics needs to be an adequate preparation for further study in mathematics for those students who will progress to level 3 study in mathematics. It also needs to be a suitable preparation for the mathematical demands of life and work for those students who will not study mathematics beyond GCSE.

This was recognised in the RSA report *Solving the Maths Problem*[3]

> *Students who struggle with mathematics in secondary schools need content that will support basic numeracy and be relevant for the workplace and everyday life. More able students need to be adequately prepared for the study of mathematics at A-level and beyond.*

The RSA report compared mathematics education in Scotland to the Linked Pair Pilot[4] in England.

> *The structure and curriculum of mathematics qualifications at National 4 and National 5 is arranged into two distinct qualifications: National 4/5 Mathematics and National 4/5 Lifeskills Mathematics. Both qualifications are of equal value but emphasise*

[1] Review of Vocational Education- the Wolf Report, 2011

[2] http://www.education.gov.uk/childrenandyoungpeople/youngpeople/qandlearning/programmes/a00225749/post-16-eng-and-maths

[3] Solving the maths problem: international perspectives on mathematics education, Norris, RSA and OCR, 2012

[4] http://www.education.gov.uk/schools/toolsandinitiatives/a00210638/mlp

different uses of mathematics. This division is similar to the linked pair GCSEs being piloted in England in that it distinguishes between functional mathematics and academic mathematics. National 4/5 Mathematics includes mainly academic maths subjects including the use of algebra (e.g. evaluating an expression or formulae that has more than one variable) and geometry (e.g. using rotational symmetry) whilst National 4/5 Lifeskills Mathematics has a greater focus on personal maths (e.g. converting between currencies) and workplace-relevant maths (e.g. comparing data sets) (SQA, 2011). National 4/5 Lifeskills Mathematics is intended as an 'exit' qualification which provides students with sufficient mathematical proficiency for the workplace and everyday situations (SQA, 2011). Because both National Qualifications are designed to be of equal difficulty and both result in the award of a National 4 or National 5 they might avoid the issues of kudos and hierarchy that have plagued prior attempts to split mathematics qualification in two in England. [5]

3.3 Ofsted survey, Tackling the challenge of low numeracy skills in young people and adults

An Ofsted[6] survey examined numeracy provision for post-16 learners in courses up to level 2. It included the following key findings.

- *Across all the settings visited, initial assessments demonstrated a high level of need for numeracy provision up to and including level 2. In some of the colleges and learning providers, more than 70% of learners started below this level.*
- *In the most successful provision, learners developed their understanding of underlying mathematical concepts through practical and vocational applications.*
- *The teaching in numeracy was more successful where providers had developed the role of one or more well-qualified and experienced numeracy specialists to support vocational trainers in planning and delivering learning sessions.*
- *The majority of the provision judged to be no better than satisfactory for classroom practice and resources focused primarily on disparate topics that were required for external tests. The individual learning plans reviewed at these providers failed to identify clear learning goals that related to the learners' personal aims and career or employment goals.*

3.4 Sutton Trust Report, The Employment Equation

A recent report for the Sutton Trust[7] had the following conclusion about mathematical competence in the workplace.

This literature review indicates that there are significant barriers to mathematical competence. It is important to emphasise that simply being able to carry out the relevant mathematical procedures is not enough. Employees need both to understand the mathematical content (though only at GCSE level) and have the capacity to understand it within their workplace. This understanding needs to include making sense of a problem, interpretation and communication. Neither the mathematical understanding nor the application capacities are sufficient on their own. Studies such as those by Julian Williams and Geoff Wake, and by Celia Hoyles,

[5] Solving the maths problem: international perspectives on mathematics education, Norris, RSA and OCR, 2012
[6] Tackling the challenge of low numeracy skills in young people and adults, Ofsted, 2011
[7] The Employment Equation, Hodgen et al, Sutton Trust, 2013

indicate that the relevant mathematical understanding is best developed through the use of problem solving techniques and a consideration of mathematics in context.

The report also highlighted the importance of tailoring mathematics content to the needs of vocational students and of maintaining the rigour and currency of GCSE Mathematics.

Finally, there are those students who don't gain a grade C at GCSE (just under half of the cohort of all 16 year-olds). As a result of the Wolf Report, this latter group will continue to study towards GCSE mathematics if they are on a school or college course. For this group, it is important to identify ways to tailor the traditional GCSE mathematics pathway to the needs of this group that are rigorous, engaging for students, provide sufficient breadth and are valued by employers.

4. Some emerging concerns about GCSE retake for vocational students

4.1 Concerns

To prepare this report, MEI has asked teachers from the FE sector about the experience of teaching GCSE Mathematics retake courses to vocational students. Some concerns have emerged from these discussions; they are noted here as possible barriers to enabling vocational students to succeed in GCSE Mathematics.

- There are teachers who have some experience of teaching Functional Skills; will their mathematics be good enough to teach GCSE?
- How can areas of mathematics which are not in Functional Skills, such as geometry, be made relevant for vocational students?
- Many students who go into FE at age 16 without GCSE Mathematics are neither emotionally nor academically prepared to retake it.
- What is to prevent colleges from offering only a Functional Skills qualification, even for students who are capable of gaining a grade C at GCSE before they are 19?
- There is a tension between trying to teach the whole of GCSE in a year and developing the understanding and mastery of basic mathematics that students will need as employees and for everyday life.

Malcolm Swan[8], writing some years ago about retaking GCSE in FE, confirmed that teachers felt a tension between covering content and developing student understanding.

The primary objective for both teachers and students appears to be utilitarian – to achieve a grade C in the GCSE examinations. Obtaining a conceptual understanding of mathematics is only a secondary objective, at least for the transmission teachers. These two objectives sometimes appear to be in conflict in the teacher's mind (eg, when considering the issue of coverage). While each teacher clearly wants to spend time helping students to develop meaning, there remains an inner anxiety that, unless students are given adequate opportunity to become acquainted (even superficially) with all of the syllabus, they will be disadvantaged in the examination.

[8] Collaborative Learning in Mathematics, Swan, 2006

4.2 Addressing the concerns

The NCETM post-16 CPD enhancement programme, which was developed earlier this year and piloted with over 20 further education colleges, has been designed to focus on those elements of mathematical subject content and subject pedagogy that enable teachers of numeracy and functional skills from the further education sector to begin the process of becoming expert teachers of GCSE Mathematics. This programme will be made available across the sector.

As a result of the new requirements, a number of further education colleges have approached MEI to lead professional development to address the needs of tutors new to teaching GCSE Mathematics. In response, MEI has developed a new programme; *Strategies for teaching resit GCSE Mathematics*. This course is designed to address the very specific needs of teaching post-16 learners working towards re-sitting GCSE Mathematics. Early feedback has shown that this course is being very positively received by both tutors and college leadership.

Some FE providers have tried using vocational contexts in teaching GCSE Mathematics to vocational students. Examples are given in the next section.

Emerging findings from a study of the experience of vocational students with Functional Mathematics show it is possible to change the negative attitudes and lack of confidence with which some students enter FE.[9]

> *The transition from school to Further Education provides an opportunity for change in students who may have previously experienced failure with mathematics. The early indications of this research are that students bring a legacy from school but it is possible to provide an environment in which student beliefs and attitudes can be reshaped, useful mathematical skills for the future can be developed and students can gain the confidence to use them.*

[9] From failure to functionality: a study of the experience of vocational students with functional mathematics in Further Education, Dalby, BSRLM proceedings, Nov 2012

5. Teaching GCSE Mathematics to vocational students

5.1 Using vocational contexts at Abingdon and Whitney College

A 2010/11 LSIS STEM case study[10] of effective practice at Abingdon and Witney College identified the problem of motivating vocational students as follows.

> *We have noticed that the motivation of many of the students for maths is low, we believe this is partly due to them finding it difficult when they were at school; they have almost all taken the exam recently and not achieved a C grade. However, we also felt that one of the key problems in motivation levels was that they did not see the relevance of the maths they were doing to their chosen vocational courses. Even if students did recognise that the GCSE maths is of some importance to them, this recognition was not enough to maintain their motivation for a year-long course.*

One group of science students at the college were also taught mathematics by their science teacher who incorporated contexts from the science course into the teaching of mathematics. Other vocational groups had material which linked to their vocational course incorporated into their mathematics lessons. Abingdon and Witney College reported on the impact of the changes as follows.

> *We will continue with this approach as it has shown some promise, although it is hard to get linked vocational material for all subjects, so it is likely to take time to build up these materials.*
>
> *Perhaps the clearest message from the case study is that having a maths lecturer who also teaches in the vocational area is a distinct advantage for the students. That said there does seem to be an improvement in the motivation of students when a non-vocational maths lecturer tries to make the maths they are studying more relevant, but this is a difficult task.*

[10] Abingdon and Witney College: Vocational Context for Mathematics, LSIS STEM, 2010/11

5.2 Wirral Metropolitan College: good practice in GCSE Mathematics

Wirral Metropolitan College is a medium sized general Further Education College. At the college, the practice for the past few years has been to encourage vocational students to retake GCSE Mathematics if they do not already have grade C or better. Wirral Metropolitan College does not offer A levels.

Students undergo a short diagnostic assessment which, together with their previous grade, enables the college to decide whether to offer them a place on a Functional Mathematics or GCSE course. Students have typically sat GCSE Mathematics at school more than once and a small, but substantial, minority have not attended school in either year 10 or year 11 or both.

To support students to succeed the college has developed the following key features in its mathematics teaching.

1. Each student completes a **short** diagnostic assessment which is built around common misconceptions in number, percentages, decimals, fractions and proportion. This informs schemes of work and lesson plans for the first weeks.
2. Induction activities focus on career goals, the mathematics required for vocational success and getting to know students' aspirations and previous experience of learning mathematics.
3. Frequent formative assessment enables tutors and learners to focus on what students need to learn; there is insufficient time to re-teach the whole GCSE course.
4. Early intervention (by week 5) and additional support (drop-in at the Maths Café) when learners aren't making progress together with close links with vocational tutors for further support.
5. Teaching mathematics through active learning, problem solving, addressing key misconceptions and encouraging cooperative learning and peer support in the classroom.
6. Contextualising problems to the vocational areas which students come from whenever possible. Where logistics allow, groups for allied curriculum areas are timetabled together e.g. Health & Social Care with Childhood Studies; Construction with Engineering.

Pass rates for Wirral Metropolitan College are above national benchmarks[11]. Students expected to succeed in GCSE Mathematics this year come from a number of curriculum areas: Art & Design, Health & Social Care, Construction, Sport Science, Business, Catering and Hair and Beauty.

The college celebrates student success both on a small scale in class and on a larger scale at their annual awards. They always nominate a Student of the Year from the 16-18 year old vocational GCSE Mathematics students.

[11] 6.2 % above benchmark in 2011/12.

5.3 Wirral Metropolitan College: contextualising mathematics for students from Catering and Hair and Beauty

Tutors at the college feel that it is important to introduce, as early as possible, the intrinsic underpinning role that mathematics plays in vocational areas. The example that follows is from two vocational areas: Catering and Hair and Beauty.

In the induction for new students, tutors asked the catering students to make minestrone soup for three people using a recipe for two people. The activity involved weighing, measuring, cutting vegetables into specific shapes, calculating time etc.

The students also had to divide a batch of dough into seven equal sized rolls by first estimating and then using some trial and error.

During their induction, the beauty students made face masks, aromatherapy oils and bath bombs with detailed and specific ratios.

The activities were followed up with a discussion about the mathematics that was involved in their chosen area and each student wrote his/her own individual learning plan detailing, not only the things he/she needed to learn, but also the topics he/she would want to learn. As a result the students were more engaged with the need to have a competent and confident relationship with mathematics. During lessons, care was also taken to consistently draw on real life examples from the vocational areas to reinforce the link.

This positive attitude towards maths needs to be fully embraced and supported by the vocational tutors, many of whom still need to exorcise their own mathematics demons. Two vocational tutors (a beauty therapy lecturer and a catering lecturer) took and passed GCSE Mathematics in one of the college's adult evening classes. They have become ambassadors and have encouraged and spurred their own vocational learners on to not give up and to successfully retake GCSE Mathematics.

Having a strong link with the vocational tutors helps to reinforce the network of support needed to enable a student to succeed. Where there are issues of attendance or lack of independent work, the vocational tutor will also step in and provide the necessary support to address these.

Note: The mathematics induction programmes for Catering and Hair and Beauty were developed as part of an LSIS STEM Funded Action Research Project. The learning materials, ILPs and materials used in the CPD process are available from heather.aspinwall@wmc.ac.uk

6. The use of context in teaching mathematics

Contexts are often used in the teaching of mathematics. Different ways in which contexts can be used in mathematics teaching are listed below.

a. A realistic problem for students to solve, using skills they have already acquired.
b. A realistic problem for students to solve in order to motivate and facilitate the learning of new skills.
c. A realistic context to enable the students to see the point of the mathematics they are learning.
d. A realistic context to help students make sense of abstract mathematics.
e. A pseudo-context which looks as though it refers to real-life at first sight but does not.

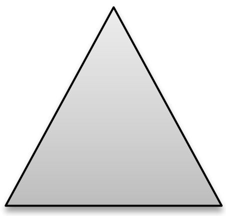
These ways of using contexts in mathematics need not be mutually exclusive but some resources, and teachers, focus more on some of them than on others. The approaches are outlined and exemplified below.

6.1 Realistic problems on which to practise skills

Many mathematics texts include such problems, often at the end of an exercise. This tends to reinforce the impression that working in a real context is more difficult than working with abstract mathematical techniques and is only for able students who finish all the other work.

An example problem which could occur at the end of a section about locus

The diagram below shows the plan view of a display stand at an art exhibition.



Scale: 2 cm to 1 m

A fence will be put around the display stand to stop visitors getting too close.
The fence will everywhere be exactly 1.5 m from the edge of the display stand.

Draw the fence accurately on the scale diagram.

6.2 *Realistic problems to learn new skills*

There has been an increased interest in problem based learning recently, at university as well as at school level. Students start with a problem; they may already know some of the mathematics needed to solve it and will also need to learn new mathematics. In addition to learning mathematical techniques, students also develop problem solving skills by working on such tasks.

An example problem to enable students to learn new skills

The image below is from a resource produced by the Royal Statistical Society Centre for Statistical Education to exemplify the teaching of statistical problem solving at GCSE level.

Source: Problem Solving Approach www.rsscse.org.uk, extracted 18/07/2013

6.3 *Realistic contexts for motivation*

In the example below, the rest of the worksheet (for carpentry students) is about working out areas of different shapes and makes no further reference to the context; the purpose of the context is to let students know why they are learning the mathematics.

An example of a realistic context for motivation

How to

... work out area

Area is the measure of a flat (two-dimensional) surface or a space covered by an object, such as floorboards or plasterboard. It may be used to work out, for example:

- how many floorboards would be needed to cover the floor in a room
- the number of plasterboard sheets needed for a stud wall partition.

Area is measured in square units, e.g. 5 square metres, or 5 m^2. The ^{2} stands for 'squared'. An easy way to remember this symbol is to think '2' means length x width (2 dimensions squared).

6.4 Realistic contexts to facilitate understanding

The purpose of the context in the example below is to build students' understanding of ratio. The context is one which many students find familiar but it is capable of being generalised into a model for dividing in a given ratio by means of sketching a rectangle, dividing it into parts and attaching a value to each part.

An example of a context to facilitate understanding

9 Kate and Pam decide it is much easier to cut up a rectangular pizza than a round one. So the next week they order the rectangular cheese and tomato pizza shown here. This pizza costs £7.20.

Kate thinks she will want about five pieces, while Pam only wants four. So they decide to cut the pizza into nine equal slices.

a) Draw a picture to show how to share out the pizza.

b) Share the cost of the pizza between Kate and Pam in the ratio 5 : 4.

Making Sense of Maths, Hodder, © Paul Dickinson, Stella Dudzic, Frank Eade, Steve Gough, Sue Hough

A team from Manchester Metropolitan University have recently been trialling some of the Making Sense of Maths resources with post-16 GCSE retake students. Emerging findings are that some students welcome the new approaches to learning mathematics which they have previously failed to understand but that the resources need to be adapted for the FE context.

6.5 Pseudo-contexts

The question below illustrates a type of question which is only seen in mathematics lessons or quizzes.

An example of a mathematics question using a pseudocontext
Bill is three times as old as Ben. In two years, Bill will be twice as old as Ben. How old are Bill and Ben now?

Some people enjoy puzzles like these but most of them will probably pass GCSE Mathematics at the first go.

Olive et al, writing in Mathematics Education and Technology: Rethinking the Terrain[12] say the following

> *These "pseudo-contexts" can actually make mathematics more difficult to learn (especially for lower socio-economic group students), as they give children conflicting messages about whether unintended contextual and experiential factors should be ignored or drawn into play. By oversimplifying the intellectual demands required to mathematize and interpret problems, and by trivializing the contribution of mathematics to solving real problems, the perception of mathematics as a subject with limited use outside of school is reinforced.*

7. Contextualising GCSE Mathematics content

7.1 Context and pedagogy

There can be a perception that working in context can make it more difficult for students to understand the mathematics but the experience of teachers who have worked with vocational students by using meaningful contexts suggests otherwise. However, it is important to choose contexts carefully – contexts which are motivating and interesting for some students will bore others. It is also important for teachers to know what learning the context is intended to facilitate; this will enable them to communicate appropriately with learners and to maximise the learning opportunities.

7.2 Context in Functional Mathematics

Functional Mathematics questions are set in a realistic context so it is natural for teachers preparing students to sit Functional Mathematics tests to teach mathematics in context. However, it is by no means straightforward for teachers to source high quality teaching materials set in suitable contexts. The Wolf report[13] describes the problem as follows.

> *The idea is that English and mathematics (and IT) should be 'embedded' in real life examples that are related to the vocational course that someone is studying and to*

[12] Mathematics Education and Technology: Rethinking the Terrain: The 17th ICMI Study, Springer, Ed: Hoyles and Lagrange, 2010
[13] Review of Vocational Education- the Wolf Report, 2011

'real life'. This is actually very difficult to do, because it demands that the teacher of the subject knows a great deal about a wide range of contexts, and can develop high quality materials for each.

Some of the best pedagogy achieves it, but as a recipe for a mass system it is highly ambitious and demanding. The alternative to having specialist teachers grapple with multiple contexts is to 'embed' the teaching in the vocational classes. That way, as we have discovered in a number of previous occasions, they embed to the point of vanishing.

Vocational teachers know about vocational subjects. They are not maths or English teachers. And if teaching maths and English were so easy that they could just be slipped into other lessons as an extra, why would so many young people be struggling with the subjects and failing their GCSEs?

If teachers do succeed in using a wide range of realistic contexts and making lessons relevant to students' other courses they will, in any case, then hit a major problem. They may be delivering large numbers of different contexts in different styles. Their students are taking a central examination, with one set of questions, the same for every candidate.

7.3 Context in GCSE Mathematics

The majority of questions in current GCSE Mathematics examinations are not set in context. An exception to this is the Applications of Mathematics GCSE, one of the Pilot Mathematics Linked Pair[14].

An example Applications of Mathematics GCSE question (part)

Some companies check employee absence using the Bradford Factor.

Bradford Factor = $S^2 \times D$

where S = number of times absent in any 52 week period
and D = **total** number of days absent in any 52 week period

(a) Judy and Dave work for a company.
They each had a total of 18 days of absence during a 52 week period.
Judy was absent twice, each time for 9 days.
Dave was absent nine times, each time for 2 days.

Work out the Bradford Factor for Judy and the Bradford Factor for Dave.

(b) Companies find that employees who have frequent short term absence disrupt work processes more than employees who have fewer times, but longer periods off work.

Describe the effect on the Bradford Factor of frequent absence.

Jan 2013, ©OCR

[14] http://www.education.gov.uk/schools/toolsandinitiatives/a00210638/mlp

The evaluation of the Mathematics Linked Pair (MLP)[15] reported as follows.

> *Centres offering both the MLP and the new single GCSE found MLP students to be more engaged with and committed to mathematics than those doing the single GCSE. The applications of mathematics, and financial applications in particular, was cited as the main reason for enhanced student engagement with, commitment to and understanding of mathematics.*

For teachers preparing students for GCSE Mathematics, there is a tension between teaching them how to answer the kinds of questions they will encounter in GCSE Mathematics examinations and teaching students mathematical skills which they will recognise as important in their future life and work. It is intended that for GCSE Mathematics for teaching from 2015 there will be more questions set in context, however, the contexts will be general rather than focused on a particular vocational area.

8. A randomised controlled trial to test a model of mathematics instruction for vocational students[16]

The National Research Center for Career and Technical Education in the United States published a report describing the results of a randomised controlled trial of a model of mathematics instruction for students enrolled in career and technical education (CTE). The study arose from concern about students' lack of mathematics skills preventing them from succeeding in future work or college courses.

The students in the trial had already been taught the mathematical content they needed in middle school and high school and might not otherwise have been studying mathematics in their CTE course.

8.1 Research questions and outcomes

The study aimed to address the following research questions.

> *1. Does a math-enhanced CTE curriculum improve math performance as measured by traditional and applied tests of math knowledge?*
>
> *2. Does enhancing a CTE curriculum reduce the acquisition of technical skills or knowledge?*

The outcome of the trial was as follows.

> *After one year of exposure to the math-enhanced lessons, the students in the experimental classrooms performed significantly better on two of three tests of math ability. Furthermore, there were no differences in measures of occupational or technical knowledge, meaning that CTE students' math skills increased without detracting from the content skills learned in their CTE courses.*

[15] The independent evaluation of the pilot of the linked pair of GCSEs in mathematics (MLP): Second Interim Report, Alpha Plus, 2012
[16] Rigor and Relevance: A Model of Enhanced Math Learning in Career and Technical Education, National Research Center for Career and Technical Education, University of Minnesota, 2007

8.2 Process and pedagogy

The trial classes made use of a three step process to enable students to make links between mathematics concepts and real problems.

- Solve a real, relevant problem.
- Practise solving related contextual problems.
- Apply the mathematical concept(s) to more abstract questions.

A mathematics pedagogy with seven elements was employed to facilitate this.

	Element	Exemplification: a lesson on gradient for construction students[17]
1	*Introduce the CTE lesson.*	Discuss flights of stairs – have they ever felt as if they were about to fall? The ideas of riser, tread and slope for stairs.
2	*Assess students' math awareness as it relates to the CTE lesson.*	Assess students' prior knowledge of rise, tread and slope through discussion.
3	*Work through the math example embedded in the CTE lesson.*	Students are introduced to the standard specifications for riser, tread, landing and headroom on stairs. They calculate the slope of real staircases and comment on how they feel to walk on.
4	*Work through related, contextual math-in-CTE examples.*	Students work with blueprints to calculate the pitch of roofs.
5	*Work through traditional math examples.*	Students calculate gradients of lines, relating their work to stairs.
6	*Students demonstrate their understanding.*	Students design a safe set of stairs for a particular specification.
7	*Formal assessment*	In this case, the assessment was to examine a set of stairs at a job site but other lessons have a short mathematics test.

Although the lessons were taught by vocational teachers, each worked with a mathematics teacher partner when preparing to teach the lesson. Some of the students were also enrolled in mathematics courses. A trial teacher describes the experience of one student as follows.

> *One day [a student] remarked that she was flunking math. I asked her why and what she was struggling with, because she did so well in the Math-in-CTE assignments. She said they were "different." I began helping her with her math assignments and related them back to what we had done in CTE. She started seeing the connections and now has an "A" in her math class.*

[17] http://www.nrccte.org/sites/default/files/lesson-plan-files/arcon_cons_lesson_01.pdf

9. The mathematical content in vocational education in other countries

ICME, the International Congress on Mathematics Education, has a study group for Mathematics Education in and for Work. This highlights the international interest in mathematics for vocational students.

In a 2003 report, West and Stedman[18], identified three main types of vocational education in other countries.

- Apprenticeships
- Including vocational options within an otherwise general curriculum
- Full-time vocational education

Some countries, including the United Kingdom, have more than one type of vocational education.

The Nuffield report, *Towards Universal Participation in post-16 Mathematics*[19] concluded that, in the countries studied, vocational education generally had a lower status than general education. However, the report also concluded that vocational education was a respected option in both Singapore and Germany.

9.1 Germany

The mathematics in vocational education in Germany is outlined below

- *80% of students in upper secondary education follow a route involving at least some vocational education (vocational schools with combined general education).*
- *20% of students follow the general education courses (Gymnasium).*
- *All students in 'technical' vocational and 90% of other vocational students are required to take mathematics. Some students of vocational education, such as engineering, also study advanced mathematics.*[20]

> *Vocational courses involving mathematics are tailored towards the individual requirements of the associated profession, and as such will range from everyday uses of mathematics, to advanced mathematics, for subjects such as engineering. Students undertaking courses leading to the Abitur examination will cover: calculus, vector space, analytic geometry and probability and statistics. Where possible the mathematics should be connected to the vocational course.*[21]

9.2 Singapore

The mathematics in post-16 vocational education at the Institute of Technical Education (ITE) in Singapore is outlined below.

> *The ITE offers four certification levels – Nitec, Higher Nitec, Master Nitec and Technical Diploma. The first two certification levels are for secondary school leavers. Nitec training courses require completion of GCE 'N' or 'O' as an entry requirement*

[18] Finding our Way: Vocational Education in England, West and Stedman, 2003
[19] Towards Universal Participation in post-16 Mathematics, Hodgen et al, Nuffield, 2013
[20] Towards Universal Participation in post-16 Mathematics, Country Profile: Germany, Hodgen et al, Nuffield, 2013
[21] ibid

with pre-requisites for certain courses while Higher Nitec training courses require GCE 'NA'[22] or 'O' with pre-requisites as an entry requirement, i.e. the entry requirements for Higher Nitec courses are higher compared to those for Nitec courses. A course of study in ITE is made up of a series of modules and there are four different types of modules – core, specialisation, life skills and elective. Students must pass all core modules and take specialisation modules in their courses in order to obtain full certification. Life skills modules focus on personal competence which an individual can transfer from one job to another while elective modules provide students with more breadth and depth in their field of study. Mathematics modules (e.g. Technical Mathematics, Mathematics 'O') in Nitec courses are offered as elective modules. In Higher Nitec, some mathematics modules are core and some are electives in a course (e.g. Engineering Mathematics is a core module and Calculus is an elective module in Process Plant Design); statistics is offered as an elective module and accounting is offered as a core in some of the courses in Business and Services.[23]

10. Vocational education in England

There is a wide variety of vocational courses in England. There are also many qualifications; West and Stedman[24] identified 2015 different vocational qualifications approved for use by students under 18 in 2003.

The Federation for Industry Sector Skills and Standards website[25] lists nineteen bodies in its directory of Sector Skills Councils (see Appendix 1)

Each Sector Skills Council covers a variety of careers and there are many different vocational qualifications taken by students in preparation for these careers. Employers are generally unfamiliar with all the vocational qualifications related to their business. Moreover, the Wolf Report[26] highlighted the importance of flexibility in the vocational education which 16-19 year old students receive.

> *In the cohort born in 1991, 62% of employed young people changed sector in the one year interval between age 17/18 and 18/19. About 40% also changed their broad occupational level. Taking an 11-year period (1998-2008), an analysis of those in their 20s and early 30s who remained in employment throughout showed that the average such individual changed jobs 3.5 times, changed occupations 2.5 times and changed sectors 1.8 times.*

In addition to giving them important skills for life and work, GCSE Mathematics can provide vocational learners with a qualification which is widely recognised across all sectors of industry. It is important that students, families and teachers understand the transferability of GCSE Mathematics and the skills it represents so that they appreciate its value in vocational programmes.

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[22] Normal Academic, taken by 25% of students. O level is taken by the 60% of students in the Express stream.
[23] Towards Universal Participation in post-16 Mathematics, Country Profile: Singapore, Hodgen et al, Nuffield, 2013
[24] Finding our Way: Vocational Education in England, West and Stedman, 2003
[25] http://www.sscalliance.org/SectorSkillsCouncils/DirectoryofSSCs/DirectorySSCs.aspx
[26] Review of Vocational Education- the Wolf Report, 2011

Students training for some vocations, e.g. engineering, will need mathematics beyond GCSE, especially if they want to progress in their chosen professions. Programmes of study for vocational students should include any mathematics beyond GCSE which they will need in order to progress in their careers.

11. Example contexts for GCSE Mathematics

The table below gives some example contexts for the draft GCSE Mathematics content which is being consulted on at the time of writing (July 2013). It follows the order of the draft GCSE content; a scheme of work for teaching GCSE Mathematics to students would group content into sensible teaching units and bring out links between different areas of mathematics.

Some of the draft GCSE content is less important for vocational students; it is more appropriate for students who will go on to study mathematics beyond Level 2. Such content has been identified in the table below.

In addition to the GCSE subject content, assessment objectives are out to consultation (see Appendix 2). Assessment Objective 3 incorporates two of the seven employability skills identified by CBI.[27]

- Problem solving – analysing facts and circumstances and applying creative thinking to develop appropriate solutions.
- Application of numeracy – manipulation of numbers, general mathematical awareness and its application in practical contexts.

In addition to using contexts from particular vocations, financial contexts are important for all students' future lives as well as their careers. All students need to learn to make sound financial decisions. The incorporation of such contexts in a GCSE retake class is in line with the recommendations of the All Party Parliamentary Group report on *Financial Education in Further Education*.[28]

> *Financial education is most effectively delivered when it falls naturally within a students' core chosen curriculum. Where this is not the case there are many more challenges in achieving coverage of financial education in the student's programme. Most institutions have used tutorial or other enrichment time to provide the space to deliver such financial education. This can be of variable reach and coverage.*

Other contexts which are generally suitable for all vocational students include quality control, use of resources, profitability and customer satisfaction.

[27] Building for Growth: Education and Skills Survey 2011, CBI
[28] Financial Education in Further Education, All Party Parliamentary Group on Financial Education for Young People, 2012

The exemplification below is intended to be illustrative, showing what could be done. It is certainly not exhaustive. Further extensive development of suitable teaching resources would need to be undertaken in order to make this useful for teachers.

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
Number N1. apply the four operations, including formal written methods, to integers, decimal fractions and simple fractions (proper and improper), and mixed numbers – all both positive and negative	• Use of decimals for finance (pounds and pence). • Use of decimals for measurement. • Use of negative numbers for temperature. • Use of fractions for parts of an hour • Use of fractions in cookery – parts of a spoon of spice or parts of a cup of flour • Use of fractions to share/portion food socially or commercially.	A wide variety of contexts is possible here; one suggested starting point, relevant to all vocations, is the context of finance. Fractions could also be linked to reading pie charts and to expressing probabilities.
N2. apply relationships between operations, including inverse operations, using conventional notation for priority of operations, including brackets, powers, roots and reciprocals	• The importance of entering a calculation correctly into a spreadsheet.	The inclusion of working with spreadsheets is highlighted in the Sutton Trust Report, The Employment Equation[29].
N3. calculate with roots, and with integer and **fractional** indices	• Use of indices in working out compound interest. • Use of indices in working out growth of a population of bacteria. • Use of square root in formulae e.g. the Mosteller formula for body surface area ($A = \frac{\sqrt{Wh}}{6}$ where W is weight in kg and h is height in m)	Indices should be linked to their use in standard form. Square roots should be connected to the inverse process to finding the area of a square and to Pythagoras' theorem.
N4. state exactly the result of calculations with fractions, surds and multiples of π; simplify and **rationalise denominators**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.

[29] The Employment Equation, Hodgen et al, Sutton Trust, 2013

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
N5. calculate and interpret with standard form A x 10^n, where 1 ≤ A < 10 and *n* is an integer	• Small and large numbers in scientific contexts e.g. atoms, cells, astronomy, population size, tolerances in engineering, differences in performance between athletes. • Interpreting numbers in standard form on a spreadsheet when working out compound interest on a loan with a high APR.	
N6. apply relationships between fractions and decimal representations, including recurring and terminating decimals	• Converting between fractions of an hour and decimal parts of an hour when calculating with time e.g. speed.	Connect to changing between fractions and percentages in the contexts of time, reading pie charts and interpreting statistics. E.g. 16 out of 150 customers surveyed were not satisfied with the product.
N7. apply the relationship between ratio and fractions	• Start with a mortar recipe and look at the relationship of amount of sand to amount of cement and of sand to mixture. • As above but with a simple biscuit recipe looking at butter and sugar. • Compare different cake recipes – which has a higher proportion of fat?	
N8. define percentage as 'number of parts per hundred', interpret percentages and percentage changes as fractions or decimals, and calculate these multiplicatively; **apply repeated percentage change**; and solve reverse percentage problems	• Interpreting market research statistics (see note for Number 6 above) • Exploring compound interest for a loan with the aid of a spreadsheet. • VAT and the price without VAT. • Taxation.	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
N9. interpret fractions and percentages as operators	• Quality control examples e.g. 5% of the pots are faulty. How many do you expect to be faulty when making 240 pots? • Risk applications e.g. past experience shows $\frac{1}{40}$ of customers leave without paying for their food. Daily sales are £540. How much do you expect to lose from non-paying customers?	
N10. check calculations using estimation and approximation, including solutions obtained using technology	• Special offers and price per unit displayed in supermarkets – spot the errors from a selection which includes some which are wrong. • Spot the error in a spreadsheet calculating how much stock needs to be ordered.	
N11. round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures), including simple error intervals using inequality notation	• Comparing a rule of thumb to an accurate formula e.g. $C = \frac{5(F-32)}{9}$ for converting Fahrenheit to Celsius versus "subtract 30 and halve the answer). • Discussion of how accurate the measuring needs to be in different contexts e.g. fitting a kitchen, ordering a carpet., beauty product doses, weighing for catering, weighing for chemistry. • Work out the number of carers required for a given number of children and round up.	
N12. apply and interpret limits of accuracy, including **upper and lower bounds**	• Tolerances in construction and engineering.	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
N13. apply the concepts and vocabulary of prime numbers, factor (divisor), multiple, common factors, common multiples, highest common factor and lowest common multiple.		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
Algebra A1. interpret and apply algebraic notation	• Entering a formula into a spreadsheet. • Working with a formula expressed in words and also in symbols and discussing which is easier to use. E.g. BMI, maximum exercising heart rate.	
A2. manipulate algebraic expressions (including those involving surds and **algebraic fractions**) by: • collecting like terms • multiplying a single term over a bracket • taking out common factors • expanding products of two or more binomials • factorising quadratic expressions, including the difference of two squares • **simplifying expressions involving sums, products and powers, including the laws of indices**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
A3. argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
A4. derive a formula, in order to solve a problem, then solve the formula	• Find prices for different electricity suppliers (standing charge and price per unit). Write a formula for the cost of *x* units. Draw graphs for different suppliers and make recommendations regarding which is cheaper. • Similar exercise for mobile phones – contract or pay as you go? • Compare plumbing services.	Link to use of spreadsheets.
A5. **understand and use function notation**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
A6. **express composition of two familiar functions using function notation**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
A7. find the inverse of familiar one-to-one functions (e.g. linear functions, reciprocal function, squaring) expressed algebraically	• Given a formula find the formula for the reverse calculation e.g. temperature conversion, working out VAT.	
A8. apply the conventions for coordinates in the plane and plot points in all four quadrants	• Finding and describing locations on an Ordnance Survey map.	
A9. plot equations that correspond to straight-line graphs in the coordinate plane; apply y = mx + c and the relationship between gradients of parallel and **perpendicular lines**	• See context for Algebra 4. • Slope of a pushchair ramp. • Slop of access ramps and play or sports equipment.	Gradients of perpendicular lines are difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
A10. deduce and apply equivalence between algebraic and graphical representations of linear, quadratic, cubic, reciprocal, **exponential and trigonometric relationships** A11. recognise, sketch and produce graphs of linear, quadratic, simple cubic functions, the reciprocal function y = 1/x with x ≠ 0, **the exponential function y = k^x for positive integer values of k, and the trigonometric functions y = sin x, y = cos x and y = tan x**	• See Algebra 4 for linear graphs. • Linear: modelling the relationship between price of a product and demand. • Quadratic: modelling the flight of a ball in a sporting context. • Quadratic: modelling the relationship between price of a product and profit. • Quadratic: modelling the relationship between speed and stopping distance for a car. • Reciprocal: modelling the relationship between the time taken to travel a fixed distance and the speed of travel. • Exponential: modelling bacterial growth. • Exponential: exploring compound interest. • **y = sin x, y = cos x**: modelling the height above ground against time of a person on a Ferris wheel • **y = sin x, y = cos x**: sound waves in music	Students using computer animations in their vocational context can produce some interesting displays here. Cubic graphs and the graph of **y = tan x** are difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
A12. **sketch translations and reflections of a given function**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
A13. generate terms of a sequence using term-to-term and position-to-term definitions	• Watch a film about the Fibonacci numbers • Explore some of the sequences in the Doodling Mathematically film • Explore sequences of lengths in op art images	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
A14. recognise and use triangle, square and cube numbers, arithmetic progressions and geometric progressions A15. deduce linear **and quadratic** expressions to calculate the nth term of a sequence	• Triangle: 5 players will each play a game of tennis against each of the other players. How many games are there? What if there is a different number of players? • Square and cube: connect to areas and volumes, tiling and packing. • Arithmetic: A theme park needs to decide whether to charge an entry fee and then have all the rides free or a smaller entry fee and each person pays the same amount for each ride. Compare the two systems. • Geometric: compound interest; bacterial growth.	
A16. deduce the sum of an arithmetic series, including where they arise in contextual problems	• Stacking tins e.g. 1 tin in the top row, 2 in the second, 3 in the third – how many tins in the whole pile? • Alice wants to get fitter. She buys a pedometer and walks 4000 steps the first day. She will increase the number of steps by 200 each day. How long before she has walked over 10 000 steps in total?	Include connecting square and triangular numbers to sums of arithmetic sequences.
A17. **construct and test conjectures about recursive and long term behaviour of geometric, quadratic and other sequences, including where they arise in contextual problems**	• The idea of pyramid selling including a discussion of why it doesn't work.	Link to exponential growth and decay.

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
A18. construct linear equations in one variable, and solve algebraically and approximately using a graph (including those that require rearrangement)	• Equations arising from the use of a linear formula e.g. given standing charge and price per unit for electricity and the total bill, how many units were used? (Would another price plan be cheaper?)	See Algebra 4.
A19. identify and interpret gradients and intercepts of linear functions graphically and algebraically	• Electricity prices with standing charge and price per unit. • Similar for phone charges – different pricing plans.	See Algebra 4.
A20. construct quadratic equations and solve algebraically by factorising, **completing the square and using the formula**; and solve approximately by using a graph (**including those that require rearrangement**)		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
A21. identify and interpret gradients, roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically and **turning points by completing the square**	• Modelling the flight of a ball – when is it dropping fastest?	Not all of this content can be contextualised in a meaningful way for vocational students.
A22. **solve equations numerically using systematic trial and improvement**	• Ben borrows £100 at 125% APR. He cannot pay it back. When will he owe more than £20 000?	
A23. construct and solve simultaneous equations in two variables (linear/linear or **linear/quadratic**) algebraically, and approximately using a graph	• Contexts as for Algebra 4 – when are the costs equal for plan A and plan B?	
A24. solve linear **and quadratic** inequalities in one **or two** variables; represent the solution set on a number line, **in set notation and on a graph**	• Contexts as for Algebra 4 – when does plan A cost more than plan B?	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
A25. plot and interpret graphs of functions, including piece-wise linear, **exponential and reciprocal graphs**, to approximate solutions to contextual problems such as simple kinematic problems involving distance, velocity and acceleration	• Graph distance against time for a runner in 100 m and 1000 m races	
A26. **calculate or estimate areas under graphs, and interpret results in cases such as velocity-time graphs and graphs in financial contexts**	• For a speed-time graph for a car, calculate the distance travelled. There is a trailer of C'étiat un rendezvous online.	
A27. construct, interpret, apply and connect algebraic, graphical and function representations, including in contextual problems	• See previous examples in Algebra 4, 10, 11.	Function representation is more formal.
Ratio, proportion and rates of change R1. use ratio and scale factor notation, including 1: r where r is a rational number, and apply methods involving conversion, mixing, measuring, scaling, comparing quantities and concentration	• Interpreting scales on maps and scale diagrams e.g. architect's drawings. • Converting between different currencies. • Scaling up a recipe from 4 people to 10 people. • Ratio of staff to patients or children. • Preparation of solutions/dilutions in chemistry. • Gear ratios in engineering.	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
R2. compare lengths, areas and volumes using ratio notation and scale factors and make links to similarity	• A doll's house maker makes a model of (local room/building) for a show. Compare lengths, areas, volumes of model and real room/building. • Look at speed of animals data. What is a good way to compare speeds of different animals, taking account of size? • Enlarging artwork or photographs – compare lengths and areas.	
R3. **construct** and interpret equations that describe direct and inverse proportion		Connect to linear and reciprocal graph examples.
R4. recognise and interpret graphs that illustrate direct and inverse proportion	• Speed v time; for fixed speed. • Speed v time; for fixed distance.	
R5. apply the concepts of speed, unit pricing and density using compound units	• Compare prices for different multi packs. • Check supermarket price per unit labels. • Work out speeds for different races in sport.	
R6. **interpret the gradient at a point on a curve as the rate of change, and apply the concepts of instantaneous and average rate of change in graphical representations (chords and tangents)**	• Interpret graphs of time series e.g. when were sales growing fastest?	
R7. apply percentage change, including percentage increase, decrease and original value problems, simple interest in financial mathematics, and repeated growth	• See contexts for Number 8.	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
R8. solve growth and decay problems, including **compound interest and use iterative processes**	• Half-life of drugs in the body. • Compound interest. • A disinfectant kills 99% of germs. Bacteria can double every twenty minutes. How long before the 1% of bacteria left after disinfecting have increased to the level before disinfecting? • Chlorine in a swimming pool reduces by 15% a day. The ideal concentration is 1 to 2 parts per million. 3 parts per million is still safe for swimming. How often does chlorine need to be added to the water?	Link to graphs.
R9. apply the concepts and vocabulary of ratio, direct and inverse proportion and rates of change, represented graphically and algebraically.	• See previous applications Ratio 4, 5.	
Geometry and measures G1. apply the concepts and vocabulary of mass, length, time, money and other measures	• Costing a business project including deciding how long it will take to complete. • Designing a kitchen using standard units to fit into a given space using scale drawing. • Designing a workplace. • Designing a play area. • Designing packaging.	
G2. derive and apply formulae to calculate: • perimeter and area of: triangles, parallelograms, trapezia, circles, and composite shapes, and • surface area, cross-sectional area and volume of: cuboids (including cubes), prisms (including cylinders), spheres, pyramids, cones and composite solids	• How many tiles to cover a wall? • Packing boxes e.g. shoe boxes or standard boxes for moving house, how much space do they take up? • How much paint to paint a room? • Carpeting floors. • Edging a flower bed or fencing a garden.	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
G3. measure line segments and angles in geometric figures, including interpreting maps and scale drawings	• Interpreting maps and scale drawings. • Decide whether a set of kitchen units will fit into a kitchen from the plan.	
G4. sketch and describe using conventional terms and notations: points, lines, planes, vertices, parallel and perpendicular lines, right angles, and regular, symmetric and irregular plane polygons	• Find parallel and perpendicular lines and symmetry in tiling patterns. • Same for stained glass or modern art. • Name shapes in tiling patterns or other designs. • Safe angles for ladders against a wall.	Make use of available software e.g. CAD, programming languages, shape drawing in word processing packages.
G5. draw and construct using mathematical tools: parallel and perpendicular lines, right angles, angle bisectors, and regular, symmetric and irregular plane polygons	• Given a stick, a rope and a metre ruler, what shapes of flower beds can be scratched into the ground with the stick (accurately)?	
G6. construct and interpret 2D representations of 3D shapes	• Construct plan and elevation drawings and match plan and elevation drawings of buildings to photos of the same buildings.	
G7. apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language	• Chris is about to start digging the foundations for a garage. He has laid out rope on the ground in a shape which he thinks is a rectangle. He measures both diagonals and finds they are equal. Does this mean he definitely has a rectangle?	Make use of available software e.g. CAD, programming languages, shape drawing in word processing packages.
G8. apply the properties of: angles at a point, angles at a point on a straight line, perpendicular lines, vertically opposite angles, parallel and intersecting lines, triangles and quadrilaterals, and interior and exterior angles of polygons	• Connect interior angles of polygons and angles at a point with tiling patterns and/or patchwork.	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
G9. identify, describe and construct congruent and similar shapes on coordinate axes, by considering rotation, reflection, translation and enlargement (**including negative and fractional scale factors**)	• Perspective and vanishing point. • Enlarging a drawing using an enlargement grid. • Explore the transformations and symmetries in the 17 wallpaper groups. • Using reflection, rotation and enlargement in computer software to create a design.	
G10. identify, describe and construct shapes transformed by stretch parallel to an axis; identify invariant points and lines of each transformation	• Explore horizontal and vertical stretches of pictures using a computer. If possible, make the stretch transparent and overlay it on the original to identify invariant points and lines.	
G11. apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures	• See Ratio, proportion and rates of change 2. • Explore ideas from Haldane "On being the right size"[30].	
G12. apply angle facts, triangle congruence, similarity and properties of named quadrilaterals to derive results and **prove conjectures about angles and sides, using transformational, axiomatic, and property-based deductive reasoning**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.

30 http://irl.cs.ucla.edu/papers/right-size.html

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
G13. identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment G14. calculate arc lengths, angles and areas of sectors	• Work in a context where a design uses parts of circles e.g. stained glass, jewellery, logo design, applique. Name parts of circles and calculate lengths and areas. • Mark out a netball pitch. • Design a running track. • Design an arch or bridge. • Calculate how much metal you need to create an arc or disc.	
G15. **apply and prove circle theorems concerning angles, radii, tangents and chords, and apply them within geometric proofs**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
G16. interpret and express trigonometric relationships algebraically and geometrically	• Connect to Algebra 10,11.	Important for construction students but more difficult to contextualise for other vocational students.
G17. apply trigonometric ratios, **sine and cosine rules**, and Pythagoras's theorem in two and **three** dimensions	• Pythagoras: use of 3, 4, 5 to get a right angle in carpentry/building. • Pythagoras: use of measuring the diagonal to check whether a shape is a rectangle in carpentry/building – include images of gates with the diagonal in place. • Trigonometry: find out the safe angle for a ramp and work out the measurements for the drop from a given doorway.	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
G18. **derive and apply area =½ ab sin C to calculate the area, sides or angles of any triangle**		This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
G19. describe translations as 2D vectors	• Use of vectors to describe a journey round a track in computer game.	
G20. apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **construct geometric arguments and proofs**	• Addition and subtraction could be done in the context of a journey, as above.	The rest is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
G21. **describe the changes and invariance achieved by combinations of rotations, reflections and translations**.	•	This is difficult to contextualise in a meaningful way for vocational students – this content is more important for students who will continue with mathematics beyond GCSE.
Probability P1. record and describe the frequency of outcomes of probability experiments using tables and frequency trees	• Play a short game on a computer, record number of wins and number of times played. Does everyone have the same chance of winning?	
P2. apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments	• Give each student one of the 16 possible lists of outcomes from tossing a coin 4 times. The list should be in a sealed envelope or by email with instructions not to show anyone. Get a student to toss a coin 4 times. Who believes you could predict the outcome? Link to Derren Brown coin toss[31].	

[31] http://nrich.maths.org/6954

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
P3. relate relative expected frequencies to theoretical probability, using appropriate language and the 0-1 scale	• On average, one in ten airline passengers does not turn up. A flight with 144 seat has been overbooked. 150 tickets have been sold. How many passengers do you expect to turn up?	
P4. apply the property that the probabilities of an exhaustive set of mutually exclusive outcomes sum to one	• For a National Lottery scratchcard, use the information about the number of cards printed and the number of prizes available to work out the probability of winning each one – compare to information given online. What is the probability of not winning anything?	
P5. enumerate sets and combinations of sets systematically, using tables, grids, tree diagrams and Venn diagrams	• Use a scratch card that has come free with a magazine. Scratch off all the windows. What was the probability of winning on this card?	
P6. construct theoretical possibility spaces for single and combined events with equally likely and mutually exclusive outcomes and use these to calculate theoretical probabilities	• Get a set of non-transitive dice. For each pair, work out which one is more likely to win.	
P7. calculate the probability of independent and dependent combined events, including tree diagrams and other representations and know the underlying assumptions P8. **calculate and interpret conditional probabilities through representation using two-way tables, tree diagrams, Venn diagrams and by using the formula**	• Use real or fictional data about medical testing to explore the problem of the false positive. • Use data from a randomised controlled trial (e.g. Finckenauer's 1982 Scared Straight trial[32]) display in a two way table or Venn diagram and interpret. • Calculate the probability of a production line failing given the probabilities of individual components failing.	The use of the conditional probability formula is less helpful than an approach based on representative frequencies.

[32] Test, learn, adapt, Cabinet Office Behavioural Insights Team, 2012

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
P9. understand that empirical samples tend towards theoretical probability distributions, with increasing sample size and with lack of bias	• Fictitious data for total sales of a product and total number found faulty over 4 weeks. What is the probability of an item being faulty? • Use an online probability simulation and see how the probability changes with more data.	
P10. interpret risk through assigning values to outcomes (e.g. games, insurance)	• Is it worth buying a lottery ticket or scratchcard? • Is it worth insuring a mobile phone?	
P11. calculate the expected outcome of a decision and relate to long-run average outcomes	• Calculate expected win on a scratch card.	
Statistics S1. apply statistics to describe a population or a large data set, inferring properties of populations or distributions from a sample, whilst knowing the limitations of sampling	• For the class, what percentage are learning to drive? For boys? For girls? How does that compare to national data? • Use national statistics to compare internet use for the class to national data.	
S2. construct and interpret appropriate charts and diagrams, including bar charts, pie charts and pictograms for categorical data, and vertical line charts for ungrouped discrete numerical data	• Use real data and ask students to construct a graph that shows it well. • Interpret graphs from a data visualisation website. • Interpret graphs from a national report relating to the chosen vocational area. • Interpret child growth charts.	
S3. **construct and interpret diagrams for grouped discrete data and continuous data, i.e. histograms with equal class intervals and cumulative frequency graphs**	• Match histograms and cumulative frequency diagrams to the real data set they come from.	

Draft GCSE content (**Bold type** identifies content for higher achieving students)	**Possible contexts for vocational students**	**Notes**
S4. interpret, analyse and compare univariate empirical distributions through: • appropriate graphical representation involving discrete, continuous and grouped data • appropriate measures of central tendency, spread and cumulative frequency (median, mean, range, quartiles and inter-quartile range, mode and modal class)	• Compare distribution of women's wages to men's wages. • Work with a small sample of wage data and find the mean by sharing out the total wage; find the median by having students holding cards with the wages on stand in order. • Use data arising from the chosen vocational area and interpret.	
S5. describe relationships in bivariate data: sketch trend lines through scatter plots; **calculate lines of best fit**; make predictions; interpolate and extrapolate trends	• Use population data – what do you think will happen next. • Predict the next gas/phone bill from real data over time. • Explore the relationship between height and arm length. • Find time series related to the chosen vocational area and interpret.	

12. Conclusions and recommendations

- In addition to giving them important skills for life and work, GCSE Mathematics can provide vocational learners with a qualification which is widely recognised across all sectors of industry. It is important that students, families and teachers understand the transferability of GCSE Mathematics and the skills it represents so that they appreciate its value in vocational programmes.

- Students training for some vocations, e.g. engineering, will need mathematics beyond GCSE, especially if they want to progress in their chosen professions. Programmes of study for vocational students should include any mathematics beyond GCSE which they will need in order to progress in their careers.

- The use of vocational contexts in teaching GCSE Mathematics to retake students can enable them to re-engage with mathematics which they have previously found difficult. In addition to increasing their chances of GCSE success, this can also improve their ability to use appropriate mathematics at work.

- Introducing mathematical content to retake students through the use of contexts allows assessment for learning to take place through discussion of what students already know and so enables teaching to focus on what students need to know.

- Some GCSE Mathematics content lends itself to a wide variety of vocational applications; other content has fewer vocational applications. At the beginning of a vocational course, students should encounter contexts which are relevant to their chosen vocational study but they should also expect to work in a wider variety of contexts during the course.

- High quality teaching resources need to be developed for vocational students retaking GCSE Mathematics. Publishers will only develop resources which will sell well; recent funding changes in FE make this market more uncertain for them.

- Mathematics teachers and vocational teachers should be encouraged and enabled to work together to explore appropriate contexts for teaching mathematics to vocational students.

- Lecturers in FE need access to GCSE Mathematics CPD. Appropriate CPD is also needed for teachers in schools and sixth form colleges who will be teaching post-16 GCSE retake students, to help them adopt teaching strategies which will enable their students to succeed.

- CPD for teachers of post-16 GCSE retake students should enable them to teach GCSE Mathematics with confidence and introduce them to ways of using contexts in their teaching, in order to improve student engagement and understanding.

Appendix 1

Sector Skills Councils

- Asset Skills (facilities management, housing, property, cleaning and parking)
- Cogent (chemicals, pharmaceuticals, nuclear, oil and gas, petroleum and polymer industries)
- ConstructionSkills (construction)
- Creative & Cultural Skills (craft, cultural heritage, design, literature, music, performing and visual arts)
- e-skills UK (software, internet and web, IT services, telecommunications and business change)
- Energy & Utility Skills (gas, power, waste management and water industries)
- Financial Skills Partnership (finance, accountancy and financial services)
- The Institute of the Motor Industry (retail motor industry)
- Improve (food and drink manufacturing and associated supply chains)
- Lantra (land management and production, animal health and welfare and environmental industries)
- People 1st (hospitality, leisure, passenger transport, travel and tourism)
- SEMTA (science, engineering and manufacturing technologies)
- SkillsActive (sports, fitness, outdoors, playwork, caravans and hair and beauty)
- Skills for Care and Development (social care, children, early years and young people's services)
- Skillset (TV, film, radio, interactive media, animation, computer games, facilities, photo imaging, publishing, advertising and fashion and textiles)
- Skills for Health (UK health)
- Skills for Justice (community justice, courts services, custodial care, fire and rescue, forensic science, law enforcement, policing and law enforcement and prosecution services)
- Skills for Logistics (freight logistics and wholesaling)
- SummitSkills (building services, engineering)

Appendix 2

Draft GCSE Assessment Objectives[33]

	Assessment objectives	Weighting
AO1	• demonstrate accuracy in the use of standard techniques and recall of mathematical knowledge • interpret mathematical notation and definitions • carry out routine procedures efficiently, with precision, especially in relation to algebra and number.	35-45% At least 40% of the AO1 marks (14-18% of the overall marks) should be within questions which also assess AO2 and/or AO3
AO2	• reason and communicate accurately, using appropriate terms and correct grammar when developing a mathematical argument (e.g. deduce, justify, generalise, prove) • construct substantial chains of reasoning, especially with algebra • construct and present clear, logical, convincing arguments, ranging from informal justifications to more rigorous deductive proofs.	30-40%
AO3	• apply mathematical knowledge and reasoning, linking mathematical ideas and using mathematical modelling to solve problems • solve non-routine problems for which an appropriate solution pathway is not immediately evident • use and apply concepts and procedures from across content domains • make connections between different parts of mathematics • solve real world modeling problems that are less well defined, making assumptions and simplifications • identify variables and construct relationships between these • having formulated problems, solve them, interpreting results and checking them for reasonableness.	20-30%

[33] GCSE subject content and assessment objectives, DfE, June 2013