“Hands up if, before today, you had heard of Mathematics in Education and Industry - MEI”.

This is a question I often ask at the start of a presentation where there is a varied audience, be it in a school, a college, a university or a company. In response to this question, people who have heard of MEI may only be aware of a specific aspect of our work, and often people have misconceptions about what we do.

Given the diverse range of work MEI undertakes, it’s usually easiest to start by dispelling some potential misunderstandings by saying what MEI isn’t! MEI isn’t an exam board, MEI isn’t a ‘for-profit’ business, MEI isn’t a government agency, MEI isn’t a large organization with offices across numerous countries, MEI isn’t …

So what is MEI? We are an independent charity, committed to improving mathematics education. We employ a team of highly committed mathematics educators with an appetite for challenge and innovation, and we work to support curriculum development and professional development for teachers, and to have a positive influence on national mathematics education policy.

To help us to answer the question ‘What does MEI do?’ John Berry, one of the editors of Mathematics in School, has asked MEI to contribute a series of short articles about its ongoing work.

It seems appropriate to begin the series, which is to be entitled ‘MEI Insights: xyz’ with a quick look back at MEI’s history, our underlying culture, and perhaps most importantly, our passion for mathematics!

**History**

On MEI’s website there is a page dedicated to the ‘History of MEI’ (Ref. 1). This charts MEI’s origins, which began in the early 1960s through the work of Bertie Bellis at Highgate School in London. The motivation for his work was to help ensure sixth form mathematics education could meet the needs of other disciplines and also support careers in industry, commerce and the professions, as well as preparing students for university mathematics. The original MEI qualification was an ‘Additional Mathematics’ O level, first examined in the summer of 1965. The papers included questions on probability, statistics and computing, all of which were completely new to mathematics education at that level.

Innovation continued, often stimulated by ideas from practising teachers, and in the 1990s MEI developed and introduced the first modular A level. It was only at this point, through the considerable success of the MEI Structured Mathematics syllabus and the need to provide associated support for teachers, that MEI set up its first permanent office with 2 full-time staff.

MEI having a (very!) small number of staff continued for several years until a new innovation, an initiative to promote and support AS/A level Further Mathematics, began in 2000. As is so often the case with MEI, the initial idea came from a classroom teacher. The initiative was called ‘Enabling Access to Further Mathematics’ and a pilot programme was funded by the Gatsby Charitable Foundation. The success of the pilot was noted in Professor Adrian Smith’s report, ‘Making Mathematics Count’, and MEI was successful in gaining support from government that meant the pilot could be extended, between 2004 and 2006, into a nationally funded ‘Further Mathematics Network’, across the whole of England. At this point, the number of staff employed by MEI moved into double figures for the first time. Also during this period, stimulated by the need to provide online teaching and learning support for AS/A level Further Mathematics, MEI created a website of mathematics resources for teachers and students, which gradually evolved into MEI’s ‘Integral’ virtual learning environment (Ref. 2).

Alongside the main Further Mathematics project, in the early 2000s work began in other areas. These included a major new professional development programme,
‘Teaching Advanced Mathematics’ (TAM) and the third edition of MEI’s A level textbooks, published by Hodder.

In 2009 the Further Mathematics Network evolved into a new government-funded programme, the Further Mathematics Support Programme (FMSP), still managed by MEI.

In 2010 Roger Porkess retired from his role as MEI Chief Executive, having led MEI for 20 years. Roger had seen MEI grow from an organization with no staff, to one with over 20. Under Roger’s leadership, MEI became a significant contributor to mathematics education at a national level. Roger was succeeded as Chief Executive by Charlie Stripp in September 2010.

Since 2010 MEI has continued to grow its work and increase its capacity to develop and provide extensive support in mathematics education. The FMSP’s work has been extended and expanded and MEI’s work in supporting both curriculum development and teachers’ professional development has grown significantly. This work will be discussed in future articles.

Culture and Love of Mathematics

As evidenced from the brief History section, recurring characteristics of MEI’s work are innovation, being at the forefront of developments in mathematics education and, perhaps most importantly, listening and responding to input from working teachers and lecturers.

We have been very fortunate that, when MEI has advertised for new members of staff, we have been able to attract such talented and committed applicants. This has meant that as the number of MEI staff has grown, the skills, expertise and specialist knowledge within MEI has also increased, bringing great benefit to our work.

A key part of our culture is that our work should, as far as possible, be evidence-based. We aim to reflect on our work in all of the projects we are involved in, learning from feedback and evaluation to improve what we do. Many projects have external evaluations, but extensive internal review also takes place, some of which is reflected in the ‘staff publications’ page on the MEI website (Ref. 3).

Almost all of MEI’s academic staff were previously teachers in schools and colleges. All are passionate about both doing and teaching mathematics. Back in 2006, the Further Mathematics Network staff were staying over in a local hotel prior to their team meeting at the offices in Trowbridge, when, during a social game of darts, an idea for a mathematical problem began to emerge. From this was born MEI’s first ‘Maths Item of the Month’ (MIOTM). Since then, each month, as the name suggests, MEI posts an interesting problem or mathematical item on the homepage of its website. The archive of past ‘items’ is available on the MEI website (Ref. 4).

Below are examples of a few MIOTMs from across the years. We hope you might find them interesting, both for yourself and also for your students. Solutions are available via the MEI website.

December 2006:
Some positive numbers add up to 19. What is their maximum product?

July 2011:
What numbers can be made from the sum of some (i.e. at least two) consecutive positive whole numbers? e.g. 1 + 2 = 3, 5 + 6 + 7 = 18

March 2008:
Are there infinitely many triangle numbers that are also square numbers?

September 2013:
ABCD is a square. Find the angle labelled α.

February 2014:
An angle θ is cut out of a circle of card to create two sectors: a major sector and a minor sector. The two sectors are then folded to make cones. What angle θ is required to obtain the largest value for the sum of the volumes of the two cones?
Free Mathematics Materials and Resources

In addition to the MIOTM MEI also produces many teaching and learning materials that are made available to schools for free. Examples of this include those seen on our ‘support materials’ webpage (Ref. 5).

It should be clear from that webpage that MEI supports not only its own ‘courses’, but also those of other organizations, e.g. providing extensive online support materials for the AQA Level 2 Certificate in Further Mathematics (Ref. 6).

MEI, through its Further Mathematics Support Programme, also provides many hard copy materials for schools and colleges, such as mathematical posters for classrooms; two of the ‘My favourite problem is...’ posters can be seen below. The full set is available via the MEI website (Ref. 7).

Next Time

As described in the opening section, this is the first in a series of short articles about MEI’s recent, on-going and future work. It hasn’t yet been decided exactly what each article will address. Likely topics include professional development, curriculum development, use of technology, enrichment, tuition and the transition to higher education. Hopefully, these are all areas that will be of interest to many readers of Mathematics in School.

URL References

(1) www.mei.org.uk/history
(2) www.integralmaths.org
(3) www.mei.org.uk/staff-publications
(4) www.mei.org.uk/month_item
(5) www.mei.org.uk/support
(6) www.mei.org.uk/aqal2cfm
(7) www.furthermaths.org.uk/prob_solv_materials

Keywords: Curriculum development; Teachers; Students; Resources.

Authors: MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.
Introduction

Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. We work to support curriculum development and professional development for teachers, and to have a positive influence on national mathematics education policy. Throughout its history, MEI has demonstrated a firm belief in the importance of technology in mathematics education, both for increasing students’ understanding of mathematics and to prepare them for a world in which technology is used extensively for mathematical processing. This, our second article in a series entitled ‘MEI Insights’, discusses the use of technology in teaching and learning mathematics.

The Importance of Technology in Mathematics Education

There are many electronic tools that can be used for mathematics: four-function scientific and graphical calculators, spreadsheets, graphing software, dynamic geometry, programming languages, Computer Algebra Systems, and other tools that are combinations of these. In addition, there are mechanical tools such as abacuses and slide rules. This article focuses on the use of electronic tools to support the teaching and learning of mathematics. What all these tools have in common is that they can perform algorithmic processes and/or represent graphical/geometrical objects quickly and accurately.

MEI believes technology has an important role to play in mathematics education because the quick and accurate processing available through using technology, combined with its ability to display mathematical images, have the potential to develop students’ understanding of mathematical concepts. Being able to perform many accurate calculations quickly can help students understand underlying mathematical structures, and multiple representations of mathematical objects can help students make important links between them, for example by observing the effect of altering a function algebraically on the graph of the function. This is further enhanced when these links are dynamic, so students can observe changes in mathematical objects directly.

In the modern world, the majority of mathematical processing is outsourced to technology. Indeed, it is already difficult to think of a future area of study or employment in which this processing is not carried out using technology (one clear exception being mathematics teaching). In preparing students for this world it is important that their mathematical education should include developing their skills in using technology for mathematics. This does not imply that we advocate training students to use technology to ‘get the answer’, rather than developing their understanding of mathematics. On the contrary, we believe technology should be embedded within the teaching and learning of mathematics in a way that enhances students’ mathematical learning by helping them to develop their understanding of mathematical concepts and by allowing them to access more mathematical ideas and applications, particularly in the use of algorithms.

These guiding principles have driven MEI’s work on developing the use of technology in mathematics education. Much of our focus has been on ways to support teachers in integrating technology into their teaching and their students’ learning, and in developing innovative ways to embed students’ use of technology in assessment.
You can view Tom and Charlie, MEI’s Learning Technologies Specialist and Chief Executive, discussing how technology might be integrated into the teaching and learning of mathematics; how using technology in the classroom enables the use of large data sets, and why teaching students to use technology in maths is important for their future careers, in a video recorded at the 2014 MEI Conference: http://bit.ly/TBCSMEI.

Supporting Teachers in Integrating Technology in their Teaching and Learning

MEI has an extensive range of contacts with teachers through textbooks, online resources, the MEI website, professional development courses, an annual conference and links through the Further Mathematics Support Programme. All of these are used to encourage the effective use of technology in teaching and learning mathematics. The textbooks and online resources feature examples and tasks involving the use of technology to enhance the development of deeper mathematical understanding, and the professional development courses and conference allow for elements focusing on this. These courses and conference sessions have focused on the use of graphing tools, such as graphical calculators or software such as Autograph, as well as the use of spreadsheets, dynamic geometry software and other tools.

MEI is currently investing in developing more resources that exploit the use of technology to enhance the teaching and learning of mathematics, for teachers and students of GCSE, A level and Core Maths. This work will include interactive resources specifically designed for use on hand-held devices.

In recent years MEI has established itself as a GeoGebra Institute. GeoGebra is free software that integrates graphing, geometry, a spreadsheet and a Computer Algebra System (CAS) within the same software tool. We support teachers and students through various means, including:

- face-to-face and online workshops for teachers;
- the design of free teaching and professional development materials for A level and GCSE Maths teachers;
- online support for GeoGebra users;
- presentations at conferences.

You can visit our GeoGebra page on the MEI website at www.mei.org.uk/geogebra to access the free materials highlighted above, which include ‘how to’ guides and task documents, as well as exemplar GeoGebra files. These are suitable for anyone from absolute beginners to more experienced users looking for some new ideas. From this page you can also get in touch with us about any requirements you may have for using GeoGebra in teaching and learning mathematics that we may be able to help you with.

MEI also works directly with education technology companies, provided the working relationship supports the use of technology to enhance mathematics teaching, rather than promoting a particular commercial product. Currently, MEI and Casio are working with a group of 50 teachers across England and Wales to develop their use of technology in A level Mathematics, including the use of graphical calculators, spreadsheets, graphing software and dynamic geometry. The aim of this network is to develop a group of teachers who can contribute to the trialling of resources that realize the potential of technology in enhancing the teaching and learning of A level Mathematics, and can support teachers in wider networks to develop their skills in embedding the use of technology into their mathematics teaching.

The network will produce classroom activities for using technology that have been tried and tested by practising teachers and the teachers themselves will be delivering training from September 2015. Updates from the network and details of the materials produced are available at: www.mei.org.uk/casio-networks (definitely a page to bookmark!)

Innovative uses of Technology in Assessment

MEI is aware of the pressures on students and teachers to maximize their performance in assessment and the impact that this has on classroom practice. Consequently, we seek to innovate in the use of technology in assessment. We believe that having some elements of assessment that require the use of technology will have a positive impact.
on how technology is used in the classroom, and so will contribute to improving students’ understanding of mathematics, and their ability to apply it.

MEI’s current A level specification (administered by OCR) includes a number of units where technology must be used within an element of the assessment. There are two units with coursework that require the use of spreadsheets for numerical methods; a topic in one of the Further Pure units that requires the use of a graphical calculator and two units, Decision Computation and Numerical Computation, where students must use spreadsheets and/or a linear programming package in the examination.

However, our most innovative use of technology in mathematics assessment is within MEI’s ‘Further Pure with Technology’ (FPT) unit. This is an optional unit for study in A level Further Mathematics and the topic areas included are: Investigation of curves, Functions of complex variables and Number theory. The last two topics are not addressed by any of the other A level specifications but are quite suitable for study at this level, particularly when using technology to produce outputs that are then interpreted (see image for an example program that could be expected to be written).

Example program:
Define program1(m)=
Prgrm
Local i,j
For i,1,m
For j,1,m
If i^2-3*j^2=1 Then
Disp i,j
EndIf
EndFor
EndFor
EndPrgm

x = 2, y = 1, x = 7, y = 4;
x = 26, y = 15, x = 97, y = 56

A succinct paper summarizing the rationale and construction of the unit was given at the 2013 International Conference on Technology in Mathematics Teaching (Button and Lee, 2013). A more extensive overview, including useful documents such as the specification and specimen/exam papers can be found at: www.mei.org.uk/FPT

The Future of Technology in Enhancing the Teaching and Learning of Mathematics

The content document for the new mathematics A levels, for first teaching from September 2017, states: “The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS/A level Mathematics.”

This content was developed through the university-led A level Content Advisory Board (ALCAB) and provides clear encouragement from higher education for the use of technology in the A level Mathematics classroom.

For A level Mathematics, the use of Computer Algebra Systems (CAS) is explicitly excluded from the assessment, because fluency with basic algebraic and calculus skills is deemed essential to developing an understanding of mathematics that can be built upon at a higher level. However, the new A level Further Mathematics does permit use of CAS in the assessment of optional elements and MEI is currently engaged in curriculum development work in this area, building from our experience of developing the current ‘Further Pure with Technology’ unit.

The new mathematics A levels are also quite bold in their intentions for the use of technology in the statistical aspects of the syllabus content, requiring that students ‘become familiar with one or more large data set(s)’ and ‘are able to use calculator technology in the examinations that will enable them to compute summary statistics and access probabilities from standard statistical distributions.’

This should put an end to boring and often meaningless statistics questions based upon plugging numbers into formulae to calculate summary statistics, or mechanically looking up numbers in statistical tables or formulae books. Instead, this explicit requirement for access to a large data set and the technology to explore it should mean students will develop a more meaningful understanding of statistical investigation and inference.

MEI is currently engaged in curriculum development work that aims to ensure that effective use of technology will enhance the new mathematics A levels, so improving students’ understanding of mathematical concepts and statistical analysis.

Further into the future, access to computer technology during assessments/examinations is likely to become routine (this does not necessarily mean that all assessment/examinations should allow the use of computers). MEI intends to embrace the opportunities this can offer, enhancing students’ understanding of mathematics and helping them to see more clearly the enormous impact the application of mathematics has on our world.

Reference

Introduction

Mathematics in Education and Industry (MEI), through its charitable status, is committed to improving mathematics education and promotes teaching and learning through numerous strands of activity. We work to have a positive influence on national mathematics education policy and one aspect of this is through our support for curriculum development.

There will be new A levels in Mathematics and Further Mathematics for first teaching from 2017. As well as pure mathematics, both mechanics and statistics will be included in A level Mathematics; there will be no choice of content.

This, our third article in a series entitled ‘MEI Insights’, aims to help teachers to start thinking about the statistics in the new A level Mathematics and how they might teach it. The article includes information about the content*, links between different aspects of the content and some example ideas for teaching.

(*Please note that due to publication schedules this article was written in July 2015. Further information on the new A levels may have emerged between submission and actual publication of this article.)

Some Features of Statistics in the new Mathematics A Levels

The content of AS and A level Mathematics for teaching from 2017 is available online (DfE, 2014). Some of the features of this content which are dealt with in this article are listed below.

• The use of technology, such as mathematical and statistical graphing tools and spreadsheets, is expected throughout the study of the new AS and A level.

• Students are expected to have calculators which will work out summary statistics from data and also probabilities from statistical distributions.

• Students are expected to become familiar with at least one large data set before the assessment, using technology to investigate questions arising from real contexts.

• The statistics content includes the binomial and normal distributions and hypothesis testing.

Learning with Real Data

Starting to teach with real data, rather than starting with teaching techniques and considering applications later, helps students see the relevance of statistics and helps them understand how to interpret data. A few example contexts are considered below.

As we grow older, we become more interested in our blood pressure – this is a topic that is probably less directly relevant to most 16-year-olds, but they may have relatives who take tablets for high blood pressure.

There are a number of risk factors for developing high blood pressure, including age. The American National Health and Nutrition Examination Survey (NHANES) contains a wealth of health related data. Probably the easiest way for teachers to download a sample of the data is through the EEPS data zoo (see references). The data and scatter diagram (Table 1, Fig. 1) are for a random sample of 10 adults from NHANES. For students to be able to draw a scatter diagram by hand without it taking too long, sample sizes need to be small. Students will already have seen scatter diagrams like this at GCSE, and KS3, and will be able to see that there is positive correlation.

<table>
<thead>
<tr>
<th>Age (years)</th>
<th>68</th>
<th>83</th>
<th>75</th>
<th>41</th>
<th>38</th>
<th>17</th>
<th>17</th>
<th>32</th>
<th>19</th>
<th>79</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systolic blood pressure (mm Hg)</td>
<td>138</td>
<td>139</td>
<td>132</td>
<td>122</td>
<td>119</td>
<td>104</td>
<td>97</td>
<td>121</td>
<td>124</td>
<td>160</td>
</tr>
</tbody>
</table>
Most students are likely to be unfamiliar with the term ‘systolic blood pressure’ and the units it is measured in. Working with real data, like this, during the A level course will allow students to understand the wide applicability of statistical techniques.

It would be a nuisance to draw a scatter diagram by hand for a larger sample but it is straightforward with a spreadsheet. Here are two scatter diagrams (Fig. 2; Fig. 3) for a random sample of 250 adults, one showing their systolic blood pressure, the other their diastolic blood pressure, each plotted against their age.

What do you notice from the scatter diagrams for the sample of size 250?

Here are some teaching points which arise from use of this real data, these would not occur naturally for students working with small artificial data sets to prepare for an examination.

• Real data sets often have missing values or errors – it is important to be able to deal with these. Unusual data values are sometimes called outliers and they need to be investigated to check whether they are real data values or errors.
• It can be difficult to tell whether there is any correlation or not – a measure of correlation is needed; the correlation coefficient measures how close the data points are to a straight line.
• There are two measurements of blood pressure taken; only one of them tends to increase with age; this can be reported on some medical websites as “blood pressure tends to increase with age”. Using the correct data is important when investigating a statistical hypothesis.

### Binomial and Normal distributions as Models

The histogram in Figure 4 shows the heights of a sample of 122 women from NHANES. A normal distribution with the same mean and standard deviation as the data set is also shown. The normal distribution seems to fit these data fairly well.

The histogram in Figure 5 shows Quetelet’s data set of chest measurements of 5738 Scottish militiamen; the normal distribution seems to fit these data well too. However, looking at other real data such as blood pressure or weight will often reveal that the data are skewed – symmetry in data sets is not as common as we
might suppose. As well as possible lack of symmetry, the detailed shapes of the ‘tails’ of data sets also quite often depart from those of a normal distribution.

Nevertheless, the normal distribution is a common and useful model for data and so the new A level includes it. An intuitive understanding of the normal distribution enables us to make predictions about what we expect to happen. For example, we might reasonably expect that school value added data are normally distributed and we would not be surprised if half the schools were below average. These data are freely available online and can be downloaded as a spreadsheet.

The graph in Figure 6 shows the KS4 data for England for 2014 – can you explain what you see?

The value added scores are based on results in the best 8 GCSEs; the graph shows a bimodal distribution; it looks as though it could be composed of two overlapping normal distributions – the schools in the smaller ‘hump’ are nearly all special schools with small numbers of students taking GCSEs.

Another commonly used distribution is the binomial distribution. Like the normal distribution, it can be used to model real data (Dudzic, 2012) but it can also be derived using mathematics which students already know from GCSE and it connects to the binomial expansion in pure mathematics.

Imagine a 10-question true/false quiz where people might just guess the answers. It is easy to see that, on average, they would get 5 questions right just by guessing. Working out the probabilities of getting different numbers of questions correct can be built up from simpler cases using tree diagrams (Fig. 7).

For a 2-question test, the probabilities are shown in Table 2. Once students have worked out the probabilities for 3 and 4 questions, they should be able to use the pattern to obtain further probabilities. Once they understand the pattern, that is a suitable time to introduce the usual formula for binomial probabilities and explore the connections with the binomial expansion.

The probabilities can also be displayed using software such as Geogebra or Autograph.

Introducing Hypothesis Testing

Imagine that a large number of people have applied for a well-paid job. The recruiters want to get the right person so they decide that the first stage in recruitment will be to administer a simple test to all applicants. The test will have 10 true/false questions; the questions will be chosen at random from a large bank of questions which reflect the knowledge needed for the job. The recruiters have devised special software to administer the test; the software requires immediate answers so there is no time to find out an unknown answer – but applicants could guess. Where should they set the pass mark to be fairly sure that people who guess all the answers do not pass the test?

Assuming that an applicant guesses all the time leads to the probabilities shown in Figure 8.

If the pass mark was set at 8, those who got 8 or more correct would pass the test. For those guessing, the software shows that this has a probability of nearly 5.5% so, on average about 5.5% of those guessing would pass the test. The organizers might decide that this is too many and choose to have a pass mark of 9 instead. Having a pass mark of 10 would be even more sure to weed out the guessers but would also eliminate some candidates who know nearly everything. Let’s assume the recruiters choose 8 as the pass mark.

Hypothesis testing is about using a sample to make a decision about a population. In the example above, a sample of 10 questions was used to make a judgement about the knowledge which an applicant possesses.

The default belief, or null hypothesis, is that the applicant is guessing (probability of correct answer = $\frac{1}{2}$). The alternative hypothesis – which will only be accepted if there is enough evidence – is that the applicant is doing better than guessing (probability of correct answer >$\frac{1}{2}$). The pass mark or critical value is 8. Some candidates will score 6 or 7 marks, so they are doing better than the average guesser but there is not enough evidence that they aren’t just lucky in their guessing.
Many students who take A level Mathematics will go on to study subjects and work in professions which make use of hypothesis testing – they will often use software to conduct hypothesis tests themselves or read about tests conducted by others. The new A level Mathematics offers students the opportunity to develop understanding of how hypothesis testing works and what it is for before going to university.

Learning hypothesis testing using a binomial distribution allows students to develop understanding of what the process involves – there are examples of its use which students can understand such as historically for Zener tests of ESP, and currently for triangle taste tests. Once students understand the process of hypothesis testing using the binomial distribution, they should be able to understand different hypothesis tests more readily.

**Extending Hypothesis Testing**

Look back at the first scatter diagram in this article showing age and systolic blood pressure for a sample of 10 people. It is fairly clear that there is some correlation in the sample but we really want to know whether there is any correlation in the population. For small samples, there is often some correlation in the sample, even if there is no correlation in the population that the sample data came from (you can demonstrate this by selecting small samples from a large data set with no correlation and seeing how the correlation of the samples varies).

A larger sample would be less likely to give a correlation that is unrepresentative of that in the population but this isn’t always possible. Suppose the sample of size 10 is the only information available. A hypothesis test can be used to decide whether the sample provides evidence of correlation in the population. The null hypothesis would be that there is no correlation in the population – we would only stop believing this if the evidence was strong enough. In this case, we want to know whether there is positive correlation in the population so we need to know how big the measure of correlation, the correlation coefficient, would need to be for the sample in order to convince us that there is positive correlation in the population. From statistical tables, in this case the critical value is 0.5494. There would only be a 5% chance of getting a correlation coefficient as high as (or higher than) 0.5954 for a sample of 10 items from a population with no correlation. Using a spreadsheet, or statistical functions on a calculator, the correlation coefficient for the sample is 0.85627; this is more than the critical value so the alternative hypothesis is accepted; there is evidence of positive correlation in the population between age and systolic blood pressure. Hypothesis testing cannot give us certainty – a high number correct in a multiple choice test or a high correlation in a sample can happen purely by chance.

Want to learn more about using data for statistical insight?

Professor Chris Wild has made the videos from the Data to Insight MOOC available on YouTube. Links to sources of data sets are available on the MEI website [http://www.mei.org.uk/data-sets](http://www.mei.org.uk/data-sets).

**References and Links**

DfE 2014 Mathematics AS and A level Content.


Information about the NHANES survey can be found at: [www.cdc.gov/nchs/nhanes.htm](http://www.cdc.gov/nchs/nhanes.htm)

The EEPS data zoo can be found at: [www.eeps.com/zoo/](http://www.eeps.com/zoo/)

Quetelet’s data is available on DASL (Data and Story Library): [http://lib.stat.cmu.edu/DASL/DataArchive.html](http://lib.stat.cmu.edu/DASL/DataArchive.html)

School and college performance tables can be found at: [www.education.gov.uk/schools/performance/](http://www.education.gov.uk/schools/performance/)

A selection of true/false quizzes where students might just guess can be found at: [http://reverent.org/quizzes.html](http://reverent.org/quizzes.html)

Wild about Statistics YouTube channel: [www.youtube.com/channel/UCEIKp33-h_Yw0o8XATHfLCg](http://www.youtube.com/channel/UCEIKp33-h_Yw0o8XATHfLCg).

**Keywords:** A level, Statistics, Real data.

**Authors** MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.
Introduction
Throughout its 50 year history, Mathematics in Education and Industry (MEI) has been providing professional development for mathematics teachers. In recent years these have included year-long courses, often in conjunction with universities, online courses with sessions spread over several weeks, 1- and 2-day courses, and sessions at our annual conference. In addition MEI is increasingly offering support through departmental reviews, a Heads of Mathematics course and, through its work in the NCETM, developing future leaders of professional development.

In this article we take four activities from MEI’s courses and use these to illustrate the following guiding principles for our professional development work.

In all its courses, MEI believes it is important to:

- promote mathematical thinking
- present mathematics in an interconnected way
- provide resources that promote mathematical learning
- support teachers in embedding new ideas in their classrooms.

The Teaching Advanced Mathematics Course
First of all we invite you to try out the activity on page 12, which was designed by MEI to use with teachers on the year-long Teaching Advanced Mathematics (TAM) course. Full details of the course itself can be seen at: www.mei.org.uk/tam.

Read the ‘information card’ and then, using only the knowledge you would expect of a student embarking on an A level course, work through the other cards. As you do this, please think about the following in the context of your own A level classroom.

- How would you introduce this activity?
- Would you adapt it and, if so, how?
- How would you group your students to work on this activity?
- What opportunities are there for differentiation?
- What would you ask those students who finish first to do next?

And finally:
- How might an inexperienced colleague’s answers to these questions be different from yours?

*** Now stop reading and have a go ***

The 1000+ teachers who have taken part in the TAM course arrive with a wide range of mathematical backgrounds and teaching experience. Whilst all teachers enjoy working through the cards, the real benefit occurs when they subsequently discuss questions similar to the first five you were asked to think about above; in this way they are supported in adapting the ideas from the TAM course for use in their own classrooms. In addition, it is common for this to motivate teachers to think creatively, seeing such a resource as a template for introducing other topics – students don’t need to be taught what they can think through for themselves!

FRESH
The second resource included here (page 13) is taken from the activity Clocks used on the 1-day course FRESH Strategies for Embedding Problem Solving. MEI’s new suite of FRESH courses is aimed at experienced teachers of mathematics. They seek to give busy teachers inspiration and suggested solutions for specific and evolving challenges within mathematics teaching. In this title we look at addressing the renewed focus of the new secondary curriculum and GCSE Mathematics on problem solving skills.

The session begins with 12 cards. A starting prompt of “what mathematical questions could we ask?” aims to promote mathematical thinking. (You may wish to jot a few down before reading further.) After spending a short
amount of time exploring the answers to one or more of these mathematical questions we discuss:

- What problem solving skills or habits might students need?
- What problem solving skills or habits might students develop further?
- What challenges would you face using this resource with your students?
- What solutions are there to these barriers?

Teachers often identify that they need support in meeting the challenge of students pursuing different lines of thought or in creating resources that provide scaffolding. The Clocks activity comes with a GeoGebra file, (bit.ly/MEIclockangles) which supports teachers in embedding these new ideas from the course in their own classrooms.

**Core Maths**

The third resource (page 14), Making Estimations from Limited Data, is from the 2-day course Starting to Teach Core Maths. This course is for anyone teaching, or thinking about offering, this new qualification. It aims to show some of the exciting new approaches being used with post-16 students and examines some of the challenges teachers face.

This particular session is about estimation and starts by asking teachers to work through an initial question about the Isle of Wight. Whilst attempting the question several points emerge.

- How can you deal with requests from students for more detailed information about the problem?
- How accurate is good enough?
- What makes a good estimate?
- When assessing this work what should I encourage/reward?

In the discussion which follows, teachers talk about the importance of explaining any assumptions made in reaching the estimate, the balance between accuracy and the amount of time spent on the problem, and the efficiency of different approaches they have used. The discussion-based teaching approach used with the teachers, based on the Japanese ‘Neriage’ method, models the one we encourage teachers to use with their students. The discussion finally focuses on what points to emphasize when teaching estimation and how to assess such questions, concluding by examining some of the sample assessments from the awarding authorities.

**Heads of Mathematics**

The final resource (page 15) is taken from Day 1 of the Heads of Mathematics course. Designed for new and aspiring heads of maths, the course aims to equip and prepare people for the challenges of this crucial role. The first day is all about the head of maths as a leader, exploring leadership styles, their current priorities and how effective leadership flows from a coherent vision of mathematics and its teaching.

The resource is from an early session which enables participants to assess their current priorities. They are given a set of responsibilities that potentially fit within the remit of a head of maths (18 are shown in the example but more are used in the course day). Participants are asked to select the nine responsibilities they deem to be most important and then arrange these in a ‘diamond nine’. Once they have decided they then have the opportunity to discuss their selections in small groups before key themes are picked up with the whole group.

Often this activity marks the beginning of the realization that it is easy to focus on responsibilities that become urgent due to deadlines or pressure from other parties rather than on those which directly enhance the teaching and learning of mathematics in a school. From this starting point we are then able to explore what effective leadership in maths looks like and the crucial need to develop a vision for what mathematics should look like within their school.

**The Future**

We hope this article has proved insightful into the thinking behind some of MEI’s courses. With the development of the Maths Hubs network MEI is increasingly providing on-going support at a local level. To find out more about your local Maths Hub and the professional development opportunities they provide see www.mathshubs.org.uk..

If you are involved with a Maths Hub and would like to work with MEI please do get in touch via www.mei.org.uk/cpd.

---

**Keywords:** Teacher support; Professional development; A level; GCSE.

**Authors** MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.
P:(0,1) and Q:(4, 4) are adjacent corners of the square PQRS (the corners being labelled in an anticlockwise fashion).

<table>
<thead>
<tr>
<th>Information Card</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A. The length of QS</td>
<td>B. The area of the square</td>
</tr>
<tr>
<td>C. The equation of PR</td>
<td>D.</td>
</tr>
<tr>
<td>E. The length of PQ</td>
<td>F. The midpoint of PR</td>
</tr>
<tr>
<td>G. The gradient of PR</td>
<td>H. The coordinates of both points where the square meets the y-axis</td>
</tr>
<tr>
<td>I. The coordinates of R and S</td>
<td>J. The equation of the perpendicular bisector of side QR</td>
</tr>
<tr>
<td>K. The coordinates of the point where the line QS meets the x-axis</td>
<td></td>
</tr>
</tbody>
</table>
On the four clocks which do not have the 12 marked, the angles are 40°, 135°, 205° and 292.5°. Find the time or marked angle on each clock – whichever is harder!
Students’ sheet – Making Estimates from Limited Information

Initial Question
At one time it was claimed that all of the world’s population could fit on the Isle of Wight. In 2011, when the world’s population was estimated to be 7 Billion, would this claim still be true?

Hint – The road distance from Freshwater to Newport is approximately 15 km.

Some further questions

How many pieces of luggage go through Heathrow airport in a year?

A company is considering opening a child day care centre in a town with a population of 200 000. There are 100 centres in the town at present. Use a quick estimate to evaluate the market for such a centre.

Five members of a swimming club plan to raise money for charity by attempting to swim around the coast of the Isle of Wight. Instead of one person swimming the whole distance they plan to take turns at swimming stages. Is this activity feasible?

A food company is planning to bid for a franchise to supply pies to all of the English football league grounds in the top four divisions for a year. When interviewing someone to manage this project the panel asks them to estimate how many pies the company will need to produce during the year to fulfil the franchise. What would be a good answer to this question?

© Contains public sector information licensed under the Open Government Licence v3.0.
### Defining your role

**Task:** Choose nine responsibilities which are most important to you in your current role. Arrange them in the ‘diamond nine’ according to the level of importance you assign to each of them.

- Developing schemes of work for maths courses
- Dealing with parents
- Recruiting new maths teachers
- Developing a maths department ethos
- Conducting maths lesson observations
- Leading the teaching of mathematics
- Being accountable for students’ exam results
- Collecting, interpreting and acting on student data
- Being a good role model
- Work scrutiny, e.g. of students’ books
- Empowering maths teachers in the department
- Tackling poor classroom practice
- Line managing maths teachers and possibly others such as teaching assistants
- Preparing for Ofsted
- Being an outstanding mathematics teacher
- Running maths department meetings
- Managing the professional development of other members of the maths department
- Keeping up to date with changes to maths curriculum and assessment
Introduction
Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. It works to support curriculum development and professional development for teachers, and to have a positive influence on national mathematics education policy. MEI, through its national Further Mathematics Support Programme (FMSP) also engages directly with students through A level tuition, enrichment and revision. This, the fifth article in the MEI Insights series, will consider how the provision for student A level revision has changed over recent years, from almost always being physical days, to now being principally online.

A Level Revision
Once the material from an A level unit has been taught to students it is usual practice for students to engage in some revision prior to examination. In some instances there will only be revision by the students themselves, but it is also likely that the classroom teacher will engage the students in some collective revision. This may be a single lesson, a series of lessons, or a complete day of revision within the school/college. However, many schools/colleges engage with external providers and materials.

The FMSP has been providing A level revision events and materials for a number of years now and the development of this provision along with benefits and disadvantages to it, will be discussed in this article.

The prominence of ‘physical’ A level revision events
Prior to 2010 revision events provided by the FMSP were generally run as ‘physical’ days. The days would normally cover one A level unit and last either half a day or a full day. They would be hosted at a university and students (and some teachers), mainly from the surrounding area, would attend. There was usually a charge for an event, which could be up to £25 per student depending on the local situation and arrangements, i.e. if a lunch was provided, etc.

The vast majority of students and teachers who attended such days indicated that they were excellent in terms of the content, the program structure and the quality of the presenters. Students benefited in several ways including: seeing an overview of a complete module in a half day/ full day event, seeing the material presented by someone other than their usual classroom teacher and they also had the benefit of visiting a university. This last point is one that has many positives in itself – several universities that hosted events would give a tour of the university over lunch time, or have a short session about the university and courses available from a lecturer or admissions tutor. Thus students were able to ‘sample’ a university in a less formal way then say an interview day, etc.

In an academic year the FMSP would generally hold over 100 of these revision events across the country. Attendances varied from small numbers for higher applied units, i.e. Mechanics 3/Statistics 4, up to 300–400 for a Core 1 or Core 2 event. Events such as Core 1 or Core 2 although not Further Mathematics units, were used as a mechanism to highlight to students the possibilities to study an AS level in Further Mathematics in Year 13.

The transition to online A level revision events
Over time the FMSP started to develop the use of online classroom technologies in its approach to teaching and learning (as well as in teacher professional development). Further discussion of tuition and professional development will not be presented here, but details can be seen in Lee (2014).

As well as continuing ‘physical’ revision events the FMSP started to offer online events during December/ January and May/June each year to coincide with the winter and summer examination series. They were free to attend by students (and teachers) of FMSP registered schools/colleges* (*Note: It does not cost to register with the FMSP at: www.furthermaths.org.uk/register.)

The events were usually around two hours in length and were also recorded so they could be replayed at a later point in time. Session leaders would cover key points and model solutions to examination questions, emphasizing key mathematical and examination techniques. The number of students in a session would vary tremendously depending on the module, from a handful to a few hundred. Interaction by students was via the use of a whiteboard, polling and instant messaging. Instant messaging did make an important contribution to the sense of a live session and frequently allowed misconceptions held by students to be seen. The session leader could then use this to increase the overall impact of the sessions.

Considering online revision sessions, compared to physical ones, then some advantages include:
• Sessions take place outside the school day, so students do not need to miss valuable contact time with their teachers, including in other subjects they are studying.

• Courses are accessible to students across the country providing opportunities to participate in revision of modules that might not be readily available locally.

• There are no attendance, travel or teacher supply costs (teachers usually attend physical events with their students).

• Material from a whole module could be reviewed in a short, typically 2 hour, session.

Some disadvantages include:

• Some students are deterred by the lack of face-to-face contact and limited interaction.

• Some students could get worried by using a technology that they are unfamiliar with.

• The sessions were short and quite intensive.

The Online Era of A Level Revision Events

To meet the national demand for physical revision days the FMSP needed to host ‘duplicated’ events across the country, i.e. there would be several Further Pure 1 (the compulsory module for AS Further Mathematics) revision events. Online it was more efficient to schedule one event for each unit – though there would be a different event for each of the awarding bodies, i.e. AQA FP1, Edexcel FP1, MEI FP1, OCR FP1 and WJEC FP1. At ‘physical’ revision events the need to cover material from a variety of awarding boards also often led to a compromise in the content covered or difficulties in splitting up groups of students who studied the different specifications. Online events can be more easily focused to the needs of students studying for a particular specification.

In 2010/11 there were 40 online revision ‘events’ available. These were attended live by over a thousand students. The recordings were viewed over 5000 times. By academic year 2011/12 there were 70 online sessions covering modules from all awarding bodies and they were attended by several thousand students, with over 8000 additional viewings of the recordings of the sessions. In 2011/12 there were still 50–60 physical events held, but this was much reduced on the number held in the previous few years.

Student feedback

At the end of an online session students (and teachers who may also have attended) were asked to complete an online feedback form. For the 2011/12 academic year there were 717 responses with 695 of those being from students and the rest being from teachers. Slightly fewer responses (565) were received in 2012/13.

A breakdown of survey responses by the examination board they were studying can be seen in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Examination Board</th>
<th>2011/12</th>
<th>2012/13</th>
</tr>
</thead>
<tbody>
<tr>
<td>AQA</td>
<td>20.4%</td>
<td>17.7%</td>
</tr>
<tr>
<td>Edexcel</td>
<td>32.4%</td>
<td>37.9%</td>
</tr>
<tr>
<td>MEI</td>
<td>20.4%</td>
<td>20.9%</td>
</tr>
<tr>
<td>OCR</td>
<td>23.4%</td>
<td>19.3%</td>
</tr>
<tr>
<td>Unknown</td>
<td>3.3%</td>
<td>4.2%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>719</td>
<td>565</td>
</tr>
</tbody>
</table>

A good spread of responses was seen across all specifications, with not much variation across the two years.

The respondents were asked to rate the online revision session on three aspects:

1. The course content.
2. Quality of delivery.
3. Online classroom as a platform for delivering the session.

Results in Table 2 indicate that the vast majority of respondents thought that the three aspects of interest were either Excellent or Good – 95.7% for the course content, 92.1% for the quality of delivery and 92% for the online classroom, in 2011/12. Similar percentages were also found in the 2012/13 responses (93.9%, 90.4% and 90.5%, respectively). Relatively few found any of these aspects to be poor.

When asked if they would recommend the online session they attended to another student then 96% of the collective 1263 responses seen across the two years said Yes! Of the collective responses to the question ‘Do you feel better prepared for your examination after this revision session?’ then 93% said Yes.

As is often the case with technology there can sometimes be issues, often outside the control of the organizer, i.e. with an internet connection, and 12% of respondents mentioned they had suffered from some kind of issue accessing the online classroom. Though this might be considered quite high, as can be seen by the overall feedback, students (and teachers) still valued the sessions and found them to be of excellent quality.

The Present and Future – Recorded A Level Revision Sessions

In 2013 a thorough review of the provision of revision was undertaken. What emerged is that for there to be live online revision sessions, for the many modules, across
multiple specifications, involved a massive amount of time and organization. This included to schedule the events, organize the sessions and to make the registration links available to the students so they attended at the correct time. Ultimately it was felt that the ‘live’ nature of the events, and all which that brought, did not carry enough benefit over a session not being live. What followed was a transition towards providing a comprehensive ‘non-live’ provision of revision.

As well as the many administrative benefits to non-live revision events it also provided opportunity to revise the ‘whole-module’ approach that was previously used in the live events. It was evident from feedback that many students didn’t necessarily need expert revision of all topics, more so of some topics. Therefore, the non-live revision sessions would instead be topic-based and much shorter in nature. This meant that the videos on specific topics could potentially be used in the classroom as a learning aid at other times of the year. It also meant that the videos would almost certainly be viewed not only by students studying for A levels, but learners in general who searched for specific topics.

A YouTube Channel, ‘FMSP Revision Videos’, was set up to host the videos. During 2013/14 and 2014/15 these videos on the YouTube Channel had over 225,000 individual views and over 1500 subscribers to the Channel. There are approaching 300 videos currently available, with half being topic-based videos and the other half being specific exam question walkthroughs. Eight of the videos have had over 3000 views each, with these being (most popular first):

- MEI S1 - Hypothesis testing
- Edexcel FP1 - Complex numbers
- Edexcel D2 - Game theory
- Edexcel FP3 - Vectors and Matrices
- Edexcel D2 - Dynamic Programming
- Edexcel FP2 - Complex Numbers
- OCR S1 - Probability
- Edexcel D2 - Transportation Problems

It is obvious that these popular videos do not contain any Core modules. A primary goal of the FMSP is with respect to Further Mathematics and as such only the Further Pure and Applied modules have had videos produced at present. Though note the FMSP does also have wider aims to support mathematics, which can be seen at: www.furthermaths.org.uk/fmsp

### In Conclusion

It is important that educators look to keep pace with technology, making good and appropriate use of suitable technologies. That’s not just using technology for the sake of technology, but using it to enhance and develop methods to support students and teachers with their educational needs. From its inception the FMSP has acted to meet demand for providing schools/colleges with high quality A level revision events. Having begun solely with ‘physical’ events it then developed live online events as an alternative. Continually reviewing the provision meant that in recent years it was determined that moving to recordings that are topic-based could make the revision materials even more accessible. This has clearly been seen to be the case with a tenfold increase in the numbers who have viewed the videos to those that attended and viewed the live online sessions previously.

### Links

- FMSP revision advice and materials can be seen at: www.furthermaths.org.uk/revision
- The FMSP Revision Videos YouTube Channel can be seen at: http://bit.ly/FMSPRevisionVideos16

### Reference


### Keywords:

A level, Online, Revision, Technology.

### Authors

MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.
Introduction

Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. Over the last decade the organization has tripled its staff numbers to its present size of about 50 employees – this is primarily down to the creation, development and expansion of the activities delivered through the Further Mathematics Support Programme (FMSP). This, the sixth article in the MEI Insights series, will consider the work of the FMSP and how it is in a position to support teachers and schools during a period of considerable curriculum development.

Background

Following the Curriculum 2000 changes to A levels the number of students taking A level Mathematics and Further Mathematics dropped dramatically. Further Mathematics disappeared from the curriculum in the majority of state-funded schools and colleges. Universities were forced to make changes to their entry requirements and adapt their first year courses to accept students with only A level Mathematics qualifications.

In 2000 MEI received funding from the Gatsby Charitable Foundation to set up a distance learning/learning support project, which was known as Enabling Access to Further Mathematics. The aim was to provide access to Further Mathematics tuition to any student who would benefit from this, through a collaboration of schools, colleges and universities, and by making use of new technology. During this successful pilot project the Integral online resources for teaching A level Mathematics and Further Mathematics were developed. MEI pioneered the use of online tuition for A level students, using the Integral resources, providing expert tuition to isolated students who would otherwise have been unable to take Further Mathematics. From 2005 the Department for Education provided funding to MEI to manage the Further Mathematics Network, which became the Further Mathematics Support Programme in 2009.

The number of students taking A level Further Mathematics has almost trebled since the programme was rolled out nationally in 2005. The number taking AS Further Mathematics has risen even more sharply. These increases have predominantly come from the state-educated sector, see Lord and Stripp (2015). In 2004–5 less than 40% of the state-funded A level Mathematics providers in England had students taking A level Further Mathematics. In 2014–15 this proportion had grown to over 66%.

Aims of the FMSP

The remit of the FMSP has widened since its inception. Whilst increasing access to Further Mathematics...
tuition remains at the core of the FMSP; the aims are now to:

- Increase participation in AS/A level Mathematics and Further Mathematics, particularly that of girls;
- Increase capacity within schools and colleges to provide high quality mathematics teaching;
- Increase demand from students to study AS/A level Mathematics and Further Mathematics post-16;
- Support improvements in level 3 mathematics education.

The FMSP works to achieve these through supporting students, promoting the study of mathematics, supporting teachers, providing professional development, providing teaching and enrichment resources and liaising with Higher Education and working with other organizations involved in level 3 mathematics education.

The FMSP continues to develop its support programmes to meet the changing needs of students and schools.

Promoting Interest and Creating Demand

The FMSP promotes interest and enjoyment in doing mathematics to thousands of secondary schools students each year. At one-day enrichment events the FMSP provides information about the continued study of mathematics after GCSE alongside engaging activities and talks illustrating the power, usefulness and beauty of mathematics. In 2015–16 over 12000 students will have attended one of these events. In addition the FMSP organizes two major team competitions, the Senior Team Maths Challenge (www.furthermaths.org.uk/stmchallenge), which is run in partnership with the UKMT, and the Year 10 Maths Feasts (www.furthermaths.org.uk/maths-feast). Together these competitions provide an enriching activity for almost 10000 14–19-year-olds. The competitions generate lots of really interesting problem-solving materials that schools can use to enhance the learning of students.

Enrichment activities are only part of the solution and the FMSP also provides CPD for KS4 teachers to improve the student experience at GCSE in order to encourage more to consider taking mathematics at A level.

A level Mathematics and Further Mathematics continue to increase in popularity. In both 2014 and 2015 Mathematics was the most popular A level subject. However, whilst it is the number 1 subject taken by boys, it is not as popular with girls, being the 4th most popular. There is still much to be done to persuade young people, especially girls, of the importance of continuing with their studies in mathematics and convince them that the subject is interesting, accessible and useful. The FMSP is liaising with Higher Education departments to inform them of the increased numbers of students taking A level Mathematics and Further Mathematics. The aim is to get departments to be more explicit about the mathematical requirements of their degree courses and to further encourage uptake at A level by indicating to prospective students the mathematical preparation that is needed to make a smooth transition. Data from UCAS shows that an increasing number of undergraduates on STEM degrees have studied A level Further Mathematics (see Baldwin and Lee, 2014).

Over the last two years the UCL Institute of Education has carried out research for the FMSP into factors affecting girls’ choices of A levels. Prior attainment at GCSE is the most important factor. Case studies of schools and colleges with above-average female participation have also highlighted the importance of building confidence and raising girls’ self-concept through support from teachers and introducing girls to more challenging mathematics in Key Stage 4. Full details of the report and additional information in this area can be seen on the FMSP website: www.furthermaths.org.uk/encouraging-girls-maths.

Flexible Approaches to Support for Students

The FMSP is able to provide students with tuition in schools/colleges that are unable to offer Further Mathematics. Originally much of this was face-to-face tuition with FMSP tutors visiting schools/colleges. However, as many more centres are able to provide support for Further Mathematics in-house, the FMSP tuition is delivered increasingly via online classrooms. Individual students from different centres across England are combined into small online groups, which provide a more engaging experience than studying alone.

With the advancement of technology the FMSP has developed the way it supports students. In 2011 the FMSP began offering live interactive lectures for a range of A level modules to support schools in the teaching of content. These lecture courses with the associated support materials provide a means by which schools can continue to offer students a good quality Further Mathematics course but on reduced contact time. Since their launch the range and popularity of the ‘lectures’ has grown. The online tuition and lectures are inexpensive and through these any student wishing to take Further Mathematics now has the opportunity to do so wherever they go to school or college in England.

Developing Teaching Capacity

For over 10 years the FMSP has provided high quality professional development for teachers. Each year the FMSP provides the equivalent of over 5000 days of CPD. Predominantly this is enhancing subject knowledge of A level topics and developing pedagogy so that A level is taught in an engaging way, developing understanding.
The FMSP has pioneered innovative and flexible approaches to delivering CPD including online courses and blended learning. The year-long Teaching Advanced Mathematics (TAM) course features a mixture of study-days, school visits and observations, online tutorials and discussions and the opportunity to acquire masters accreditation. The aim of this course is to increase confidence in A level Mathematics content for teachers who have not taught A level before.

The FMSP has also created resources to help teachers develop and practise mathematical problem-solving skills: www.furthermaths.org.uk/prob_solv_materials.

Many of the FMSP CPD courses focus on problem-solving skills development at Key Stage 4 and A level, including the higher-level problem-solving skills necessary to tackle problems in STEP, AEA and the MAT examinations. All of these resources and CPD courses aim to support teachers with the new requirements in GCSE Mathematics and the new A levels.

Access to Higher Education

In the last two years the FMSP has begun providing information and guidance for students and teachers on the mathematical requirements of a range of undergraduate degrees. The FMSP website (www.furthermaths.org.uk/universities) has resources illustrating the mathematics that students are likely to encounter in the first year of different degree courses, as well as information about the likely entry requirements. The aim is to ensure that students are adequately prepared for the transition to degree study.

The FMSP has also established regular support in a number of areas for students from state-funded institutions who are preparing for STEP, AEA and the MAT examinations. These additional qualifications are required for entry to study mathematics at some prestigious universities and this can be an obstacle for students whose schools/colleges are not able to support them. In the longer term the FMSP’s programme of CPD focusing on higher-level problem-solving should mean that many more teachers will have the confidence to support their students with preparation for STEP, AEA and the MAT.

The New A levels and FMSP Support

Over the next 18 months teachers will be getting ready for teaching the new A level specifications for mathematics. Whilst actual specifications or sample assessment materials are unlikely to be seen until later in the summer term, the FMSP has begun providing information about the changes through its website and through teacher network meetings. These meetings provide an invaluable forum for teachers to discuss the implications of moving to a linear A level, the 100% specified content in A level Mathematics, including statistics and mechanics, and the decoupling of AS level from A level. It is less clear exactly what Further Mathematics will be like, which has 50% specified content, as there is likely to be some choice of topics. As more details about the changes emerge this year the FMSP will be producing resources to support teachers and adapting its CPD provision and support for students, see: www.furthermaths.org.uk/2017.

In Conclusion

The FMSP is a government-funded programme that seeks to support students, teachers and schools with mathematics. Local events and details of the relevant Area Coordinators can be found from the website’s regions page (www.furthermaths.org.uk/regions), and the central management team can be seen on the contacts page: www.furthermaths.org.uk/contact.

References


Keywords: A level, Professional development; Higher education.

Authors MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.
Introduction
Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. It works to support curriculum development and professional development for teachers, and to have a positive influence on national mathematics education policy. MEI, through its national Further Mathematics Support Programme (FMSP) also engages directly with students through mathematics enrichment, A level tuition, and revision. This, the seventh article in the MEI Insights series, will look at the variety of enrichment opportunities that the FMSP offers and the work that is being done to encourage teachers to include enrichment activities in their day-to-day teaching.

What is Mathematics Enrichment?
A student can experience enrichment in a number of ways; it may be through attending a large-scale event at a university and hearing about some aspect of mathematics of which they weren’t aware; it may be through looking at how the mathematics they have been taught is used in the world of work; it may be through the development of their mathematical thinking skills by solving interesting problems. The aim of any enrichment activity is to add something to a student’s mathematical experience, hopefully promoting an understanding of a wider world of mathematics of which their school studies are a part. The FMSP in its enrichment programme offers schools the opportunity for their students to engage in each of these types of activity.

The Development of FMSP Enrichment
The work of the FMSP has widened considerably since its inception and the nature of the enrichment it provides to students has evolved over the years. The first enrichment events organized by the Further Maths Network (which became the FMSP in 2009) were based around the model of having 100 to 300 Year 10 or 11 students attend a day in a university setting taking part in workshops and hearing talks about a variety of aspects of mathematics that their school studies might lead into. These events usually included an inspirational lecture by a keynote speaker.

Over the years, the range of enrichment activities by the FMSP has increased. The large-scale events still exist and form a key part of the FMSP’s enrichment programme but they have been added to with:

i. What’s the Point? shows (in partnership with Maths Inspiration)
ii. Celebrating Women in Mathematics events
iii. Mathematics at Work events
iv. Small-scale enrichment workshops
v. In-school enrichment workshops
vi. Problem-solving competitions
vii. Enrichment activities for Key Stage 3 students
viii. Enrichment materials for classroom use
ix. Enrichment and problem-solving posters for schools.

In 2015–16 over 14,000 students attended an FMSP enrichment event. In addition to this, more than 7500 students attended one of the two FMSP organized team competitions, the Senior Team Maths Challenge, which is run in partnership with the UKMT (www.furthermaths.org.uk/stmchallenge), and the Year 10 Maths Feasts (www.furthermaths.org.uk/maths-feast).

What follows is an overview of the FMSP enrichment activities listed above with a discussion of their content, organization and purpose. Thereafter follows some feedback gathered from the events and a final short summary.

1. Large-scale Enrichment Events
The FMSP’s large-scale events still make up a large proportion of the enrichment numbers and the model for these has remained largely unchanged. Although these events tend to take place in universities, a large number now use other venues such as school halls or local government rooms. These enable students in areas where travel to the nearest university is impractical to have access to FMSP enrichment.
The programme for an FMSP enrichment day typically consists of a number of workshops, where the students are split into smaller groups, and a keynote lecture. Workshops are usually interactive allowing students to take an active part in the mathematics. They may be asked to take part in a ‘true’ (the three-person equivalent of a duel) to illustrate an aspect of game theory, they may find themselves lying on a plank of wood balancing on two small wooden blocks to illustrate the principle of moments, or they may find themselves acting as the components of a giant tower of Hanoi puzzle in order to see a recursive relationship in action.

The keynote talk is usually provided by a speaker well known for delivering mathematics enrichment in an accessible and engaging way. Past keynote speakers have included Colin Wright, talking about the mathematics of juggling, Ben Sparks talking about the birth of number, Hugh Hunt talking about the mathematics of boomerangs and Coralie Colmez talking about the inappropriate use of maths as evidence in court.

The purpose of these large-scale events is not only to provide students with some insight into the use of the mathematics they know beyond their GCSE. At each event there is a short presentation that links to the enrichment talks and encourages students to consider studying Mathematics and/or Further Mathematics at AS or A level. The enrichment days therefore fill an important role in enabling the FMSP to achieve its aims to increase the uptake of A level Mathematics and Further Mathematics.

2. What’s the Point? Shows

The ‘What’s the Point?’ shows began in 2015 and are a joint venture between Maths Inspiration and the FMSP (www.mathsinspiration.com/summer-2016). The shows are designed to inspire Year 9 and 10 students, in particular the ones who don’t love maths. Each show features some of the country’s most entertaining maths speakers explaining why there is actually a point to the maths that students have been studying. The shows are planned jointly by the FMSP and Maths Inspiration, and the location of each of them is based on regional need. The FMSP also enable students from schools who would not otherwise be able to engage with the shows to attend.

3. Celebrating Women in Mathematics

One of the aims of the FMSP is to increase the proportion of female students taking Mathematics and Further Mathematics at A level. Following research into girls’ participation in A level Mathematics, the FMSP and Institute of Education have produced a report identifying the key factors (see: http://bit.ly/1LDKiWU). These include prior achievement, enjoyment, self-concept, stereotyping and the utility of mathematics. The FMSP leaflet, available on the FMSP website (www.furthermaths.org.uk/files/FMSP-Girls-in-Maths.pdf), contains strategies for promoting a better gender balance in A level Mathematics.

The FMSP runs a variety of events that challenge the stereotypical image of mathematicians and focus on the factors that the research has shown affects girls’ participation. These events are similar in nature to the FMSP’s large scale enrichment events but are tailored towards highlighting the contribution that women have made to advances in mathematics. The events are not restricted to female students only and students of all genders attend.

4. Mathematics at Work

The FMSP Mathematics at Work events focus on the use of mathematics in a variety of careers. Speakers from a variety of industries deliver presentations showing how they use mathematics in their day-to-day work. These events provide answers to the perennial student question “where would I use this?” as well as identifying what students need to be able to do if they wish to follow a particular career path. Speakers at past Mathematics at Work days have included representatives from industries as diverse as Computer Games Design, Forensics and Astronomy.

5. Smaller Scale and In-school Enrichment

The FMSP has a remit to increase participation in mathematics enrichment. Whilst the large-scale events are usually well attended, there are many schools that are unable, or perhaps unwilling, to allow students to travel to an enrichment event for a whole day. In order to enable students from these schools to experience some enrichment activity the FMSP has, over the past two years, introduced a programme of smaller enrichment events. These events are usually held at a school that is unable to send students out to the larger events. Other schools from the surrounding area may be invited to attend. In-school enrichment of this nature is flexible in nature and can be a full day, a half day or simply one workshop in a lesson. The enrichment workshops provided by the FMSP typically focus on areas of mathematics related to those the students are studying.

6. Problem-solving Competitions

The FMSP organizes two major team competitions, the Senior Team Maths Challenge (www.furthermaths.org.uk/stmchallenge), which is run in partnership with the UKMT, and the Year 10 Maths Feasts (www.furthermaths.org.uk/maths-feast). These competitions aim to provide students with the opportunity to develop their problem-solving skills in rounds that test a variety of styles of mathematical thinking and teamwork.

The Year 10 Maths Feast is a stand-alone competition and has no heats or finals. Participating schools are
encouraged to take the materials back to their schools and use them with other students in their classes to enrich and extend their learning. In this way, more students can benefit from the experience than just the students who make up a team.

The structure of some of the rounds within each competition changes each year. Rounds in past competitions have included a comprehension round, a relay and a practical round in which the students attempted to make origami Columbus cubes and build a tower with them. The FMSP website includes both the past competition materials as well as additional materials to help prepare students for the competition. These provide a guide to the standard of the mathematics and the sort of problems they might face (www.furthermaths.org.uk/maths-feast-materials).

Here are some examples of the problems from the 2015 Maths Feast Competition:

In the Entrée round, students were given a poster of information about Set Theory and Venn diagrams. After reading through the information, they were asked questions including the two below.

1. Given \( A = \{1, 2, 4, 8, 16\} \), \( B = \{2, 4, 6, 8, 10, 12\} \) and \( C = \{1, 2, 3, 4, \ldots, 20\} \),

Write down:

\[
n(A) = \quad \quad A \cap B = \\
n(A \cup B^c) = \\
A \cup B = \\
A^c \cap B = \\
\]

2. The shaded region on the Venn diagram shows \( A \cap B \).
7. Enrichment Activities for Key Stage 3 Students
In previous years, the majority of enrichment work done by the FMSP has focused on Year 10 and 11 students. This year, the FMSP has been able to extend its enrichment programme to include all 11–16 students. Events such as the Key Stage 3 Mathemagic day, which consist of puzzles, problem-solving and a quiz, are now being run across the country.

8. Enrichment Materials for Classroom Use
The FMSP has a commitment to helping teachers enrich the experience of students in their lessons and have produced two enrichment packs of materials for use in classrooms. Materials in them include specific enrichment activities that teachers can try out with their students, GCSE extension materials, problem-solving materials and Maths Feast past materials with the accompanying additional resources. A presentation and leaflet to encourage students to study mathematics beyond GCSE are also included. Details for how to obtain one of these packs can be found on the FMSP website (www.furthermaths.org.uk/enrichment_materials).

Below are some examples of what can be found in the FMSP enrichment packs.

9. Enrichment and Problem-solving Posters for Schools
The FMSP have also produced a number of high-quality posters designed to engage and enrich the experience of 11–16 students. There are two series of posters; one set highlights where some areas of A level Mathematics are used in a way that younger students can understand and the other shows a range of interesting mathematical problems of varying difficulty that are the favourites of the FMSP’s staff. Details for how to obtain these can be found on the FMSP website (www.furthermaths.org.uk/enrichment_materials and www.furthermaths.org.uk/prob_solv_materials).

Feedback from FMSP Enrichment Activities
The FMSP has a commitment to providing high-quality enrichment for 11–16-year-old students with the aim of encouraging the uptake of A level Mathematics and Further Mathematics. Feedback forms from students and teachers are collected at large-scale events and from teachers when in-school enrichment has been provided.
One of the “My favourite problem is...” postcards produced by the FMSP

FMSM enrichment posters show some of the uses of A level Mathematics and Further Mathematics
From the many thousands of feedback forms received for the current school year, the students have indicated that of the 32% who had already decided to study mathematics, 86% of these said that the event they had attended had made them more interested in studying mathematics. 45% of students who attended stated that they had initially been uninterested, but made more likely to study mathematics by the event. 88% of students indicated that they would recommend the event they attended to other students.

Here are some examples of student feedback about what they gained from attending an FMSP enrichment event:

“Maths opens a lot of doors! Very interesting facts about maths that I didn’t know before.”

“Maths has an everyday use. Maths at A level keeps your options open.”

“Very inspirational!! Definitely going to be taking further maths after today.”

“That maths is complicated but FUN!!”

“That nothing is impossible.”

“The extent that maths is being used in the real world.”

“That there is maths behind a lot of careers.”

“That maths is a wide subject and can be linked into other subjects such as geography, science and psychology.”

“Thanks for a great day, really enjoyed it and learnt a lot.”

In Summary

MEI, through the FMSP, provides students with a wide variety of enrichment opportunities. These range from highlighting uses for the mathematics they know through to developing problem-solving skills and mathematical thought. The FMSP also produces materials for teachers to use to enrich the mathematical experience of their students. Local enrichment events and details of the relevant Area Coordinators can be found from the website’s regions page: www.furthermaths.org.uk/regions.

Keywords: Key Stage 3; GCSE; A level; Enrichment; Problem-solving.

Authors Phil Chaffé and Stephen Lee, MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.

e-mail: stephenlee1643@gmail.com
Introduction

Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. It achieves this through a range of activities including curriculum development, professional development and the provision of teaching and learning resources. In this, the eighth in our MEI Insights series, we consider the Integral virtual learning environment of teaching and learning resources. This includes its origin, a look at its contents and plans for future development, particularly with new mathematics AS/A levels being introduced for first teaching from September 2017.

Origins of MEI’s Online Resources

MEI has provided online teaching and learning resources since 2001. The first resources were created to support AS/A level Further Mathematics students who participated in the pilot ‘Enabling Access to Further Mathematics’ project which ran from 2000–2005. This was managed by MEI and funded by the Gatsby Charitable Foundation. This developed into the DfE-funded national Further Mathematics Network (FMN) and then the Further Mathematics Support Programme (FMSP), as detailed in the MEI Insights 6 article.

Further Mathematics students using the resources would not necessarily have frequent access to a teacher or tutor. As such, the resources were written with a student reader in mind, containing lots of additional examples and exemplifying a good written style. Online assessment was used to provide feedback to students on their progress; assessment questions addressed both basic technique and deeper, multi-step scenarios. Initially these resources were hosted in a bespoke website under the title of ‘MEI Online Resources’.

More and more resources were created over subsequent years and there was a need to introduce new functionality, enable more staff members to upload and edit resources and keep up with the latest expectations in website design.

Development of the Integral VLE

In 2009 the set of resources was reorganized into a dedicated Moodle Virtual Learning Environment. This enabled MEI to have much greater control to further develop the content and site design in-house. At this time the site became known as ‘Integral’ and development of resources for AS/A level Mathematics started.

Figure 1 shows a screenshot of a single section of Integral. Most topics are covered by several sections like this. This section is representative; some, but not all, resource types are common to all sections.

The following are common to all sections:

- ‘Before you start…’ text
- Section overview/Notes and examples/Crucial points
- PDF documents
- Exercise questions and solutions
- Interactive/active learning resources and external website links
- Section test (taken on screen and automatically marked)
- ‘Now you have finished…’ text

To help manage their studies, students can place ticks in the boxes to the right of the resource links to record their own engagement with them.

Within the VLE there are many different file types, including: PDFs, Word documents, PowerPoints, Spreadsheets, Geogebra files, JavaScript interactive questions, videos, as well as hyperlinks to external resources (as appropriate to sites such as mathcentre). Icons are used to give an indication of the file and resource type.

A complete suite of online teaching and learning resources was created in the Integral for all awarding organizations’ AS/A level Mathematics and Further Mathematics specifications (i.e. not just the MEI specification, but also AQA, Edexcel and OCR).
Anecdotal feedback on resources of this kind suggests that the level of engagement they produce compares well to when equivalent tasks are presented in a traditional exercise format. Students have a greater tendency to look for new methods for carrying out routine calculations when tasks are presented like this. Similarly, they are effective when students are in groups, leading to helpful discussion and students supporting and learning from one another.

Not all teaching resources produced are intended for printing or display. Some provide advice to teachers on useful questions and discussions. The following questions can be used to reinforce and assess learning. Students can answer on mini-whiteboards and should be told that they need to find a different answer from anyone sitting near them.

- Give me an example of an equation whose graph has a $y$-intercept of 5.
- Give me an example of an equation whose graph has an $x$-intercept of 2.
- Give me an example of an equation whose graph has a gradient of 4.
- Give me an example of an equation whose graph passes through the point $(5, 3)$.
- Give me an example of an equation whose graph has a $y$-intercept of 5.

This kind of questioning can help students build understanding, develop mathematical thinking as well as an ability to cope with deviations from the way that they may have first practised techniques and processes.

The following YouTube video shows more examples of teaching resources available in Integral: [www.youtube.com/watch?v=0_IayfJ0Ecs](http://www.youtube.com/watch?v=0_IayfJ0Ecs)

Fig. 2 Matching teaching resource

Unit: Improving learning in mathematics (Swan, 2005). This kind of questioning can help students build understanding, develop mathematical thinking as well as an ability to cope with deviations from the way that they may have first practised techniques and processes.

The following YouTube video shows more examples of teaching resources available in Integral: [www.youtube.com/watch?v=0_IayfJ0Ecs](http://www.youtube.com/watch?v=0_IayfJ0Ecs)

Classroom resources like this are placed next to student resources on the same topic in Integral enabling teachers to plan lessons and homework for each topic effectively.
The Modern Era of Online Resources

Smart phones, tablets, apps and YouTube becoming a significant part of everyday life for many people has led to different expectations from online support for the teaching and learning of mathematics.

Students’ gathering information and learning from watching video clips is a normal practice. The FMSP produced revision videos which have been linked to from MEI’s online resources. Since uploading the first of these in April 2014 to the present time (June 2016) there have been almost 400,000 views.

Websites now offer learning experiences which have questions and tasks blended into other medium for information transmission such as text and videos. Responses to questions and tasks can be automatically checked and feedback provided. Completion of each unlocks access to the next so that the learning can be carefully staged. MEI has been developing ‘Walkthroughs’ to offer this kind of learning experience. Figure 3 shows one task from a sequence about stationary points.

The following YouTube video provides more information about Walkthroughs: www.youtube.com/watch?v=hz7RQTS_Vvc

It is interesting to think about the role of traditional textbooks in this modern era. Many textbook publishers now provide their titles in an electronic format, responsive to the screen size of any device. Integral has been working with the publisher Hodder to ensure close integration between its new electronic textbooks for A level Mathematics and Integral. This means that students and teachers will easily be able to access associated Integral resources from the relevant sections of the electronic textbook.

MEI carried out a survey of teachers accessing Integral in Autumn 2014 which asked for views on the role of textbooks. Of the 139 teachers that responded to the Integral survey 69% said that expected to be using a combination of textbooks and online resources to support their teaching for the foreseeable future. Of those who said they would continue to use textbooks, 63% expressed a preference for hard copy books.

Resources for New A levels for First Teaching from September 2017

AS/A levels in Mathematics and Further Mathematics are being revised for first teaching in 2017. There is to be greater emphasis on problem solving and reasoning and the use of technology. This includes using software with spreadsheet functionality to analyse large data sets in statistics.

MEI is working to create resources that will enable students and teachers to acquire the necessary skills with technology. This will include technology skills videos.
For more information about plans to develop Integral for the new AS/A levels in mathematics see: www.mei.org.uk/2017

Note: Full access to Integral, including for students, is by annual subscription. Prices start from £265 per year and depend on the number of student accounts required. With a subscription, student activity can be tracked and online test scores analysed. A school/college maths department can get free teacher access to all of Integral except for Core 1 to Core 4 when it registers with the Further Mathematics Support Programme (FMSP).

Closing Remark

Over the last decade MEI has developed extensive teaching and learning resources for mathematics. These have been updated in content and style in line with advancing practices, and will be of great relevance to the new mathematics AS/A levels for first teaching in 2017. The benefits of these on-going developments, through access to many AS/A level materials, are available for free to all schools/colleges through registration with the FMSP.

Keywords: A level; Technology; Integral; Problem solving.

Authors Richard Lissaman and Stephen Lee, MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.

---

Fig. 4 Problem-solving starter question

Looking for ideas?

- Extension material
- Problem solving
- Fun word problems
- Enriching the curriculum
- Challenging your pupils

Look no further.

Include PMC in your mathematics resources.
Background

Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. As part of its provision, MEI's Further Mathematics Support Programme has a commitment to provide Professional Development for teachers of Key Stage 4 Mathematics students, specifically targeting Extension and Enrichment work. The project started in 2012 and continues to be popular, with the two-day course currently being attended by a fifth cohort of teachers. In addition to the two-day course there is a selection of one-day courses which target new mathematical content from the revised Higher Tier GCSE specifications, and an annual conference. All of these events are free to teachers in state-funded schools and academies, which is made possible by continued funding from the Department for Education. To date, approximately 2,500 teachers have attended one or more of these events. This article outlines the project, discusses some of the issues and shares some of the more successful resources and approaches.

Guiding Principles

There are several aims and principles that underpinned this project from the outset.

- The intention of the resources and the professional development sessions should be to ensure that students develop a deeper understanding of GCSE content, promoting students making connections within and between topics rather than moving them on to additional content. This should serve students well when sitting their GCSE exams and give them a firm foundation to build on at A level.

- Resources should be easy to use in the classroom.

- Teachers should be encouraged to explore what works for them and their students. We are trying to find small changes and tweaks to usual practice that will add challenge for students or help to develop skills or understanding rather than seeking large-scale changes.

- Finally, when putting together sessions and activities for teachers, outcomes of research into effective professional development should be taken into consideration. The NCETM's RECME report (Back et al., 2009) still serves as a very good guide for this and several of the recommendations have been embodied in the design of the professional development sessions. A key feature of the professional development is that teachers have the opportunity to try out resources themselves and consider and discuss how they might present them to students before using them in the classroom and then feeding back to a small group of peers on the second day.

Where's the Gap?

Before putting together any resources or professional development sessions, one of the most important initial tasks was to find out what teachers wanted or felt was lacking. The outcome of a small-scale research project was a shopping list of useful features of resources, together with a selection of topics and skills that would ideally be developed. The participants for the study were drawn from 11–16 and 11–18 schools as well as from Sixth Form Colleges, and one of the most interesting things was the degree of overlap in their responses. That made things easier, as it suggested that KS4 teachers wanted to teach topics and develop skills that post-16 teachers wanted students to better comprehend or develop!

Figure 1 is a representation of all responses, with font size being proportional to the number of teachers mentioning the element.

It is perhaps not surprising that ‘algebra’ featured highly within the responses, as this is often cited as a
source of concern at both GCSE and A level, but it was interesting that teachers also came up with ‘developing understanding’ and ‘thinking flexibly’ as worth focusing on as that was one of the perceived ‘gaps’ that this project hoped to address.

What's Wanted?

When developing the first set of resources several considerations as well as requirements emerged from the research project. Some of the elements arising from interviews with teachers were:

- The resources should address the different needs within classes in schools, i.e. a whole set of students who ‘should’ go beyond GCSE or half a class of students who should be extended or a handful of students who require extension material.

- Thought should be given to the ‘barriers’ that prevent teachers from using materials such as teachers’ lack of ICT skills and amount of photocopying. Several of the teachers interviewed indicated that student access to computers in maths lessons was extremely limited and that although calculators were easily accessible, access to graphical calculators is limited.

- Materials which are flexible in their use would be welcomed, i.e. something that could be given to a handful of students within a class to extend the more able or that could be used for a whole-class starter or plenary activity.

- Resources that are quick and easy to use, with little preparation, both in terms of the resource itself and in terms of the teacher familiarizing themselves with the content would be welcomed.

Teachers indicated that within the set of resources there should be materials to meet a range of learning needs for students taking the Higher tier GCSE, thus different resources should be designed:

- For practice and consolidation, particularly in non-standard ways.

- To help teachers identify students who hold common misconceptions.

- To help students make links between and within topics.

- To develop problem-solving skills.

- To incorporate real-life contexts and modelling.

Meeting the Requirements

It would be incredibly difficult to come up with a single resource to meet all of the requirements, so a pack of resources (which between them cover the requirements) was put together.

A two-day course was devised, with days 1 and 2 being six to twelve weeks apart to give teachers time to try out some of the resources with their classes. Day 1 sessions focus on exploring a wide range of resources, thinking about what ‘extra’ is gleaned from richer resources, questioning and classroom issues. Day 2 sessions are more reflective but there are also opportunities to look at further resources and approaches.

Teachers completed short questionnaires at the start and end of the course outlining their current situation and initially indicating what they hoped to gain and then reflecting on what they actually gained. They also asked students to review a lesson using a selected resource. The summaries of these data have been used to inform subsequent iterations of the course and the design of further materials.

What Worked Well?

An intention when developing resources was that there should be examples of resource structures which are transferable so that if teachers find them useful they can create their own for other mathematical content, or indeed for other age groups or attainment levels.

One of the most popular resource structures developed is a maze. An example is ‘Algebraic Expressions 1 Maze’ which you might like to try before proceeding (Activity A at the end of this article). The objective is for students to practise their algebra skills, but it is also carefully designed to allow the teacher to pick up on misconceptions, since several common ones are targeted at various junctions. For example, the very first move would indicate any student who thought that $a^2$ is the same as $2a$ or that, when squaring, it’s just the first element of an expression that the operation is applied to. Mazes proved popular because it is obvious what needs to be done and the only preparation needed is photocopying. In fact, the vast majority of these resources are available as PowerPoints too, so they can be used easily as starter or plenary activities for whole classes. This particular one could be used to gauge the level of need just prior to returning to an algebraic expressions topic.

Another resource structure utilized is a card-matching activity. Whilst these have been popular for many years, the availability of card and jigsaw-creating software has perhaps encouraged the creation of a certain type of card
set in which there is one match for each card. One of the interesting developments from the "Improving Learning in Mathematics" project (Swan, 2005) was the creation of card sets where this was not the case, where there were blank cards or a range of valid responses. This approach seemed to be more challenging, opened up discussion and allowed students and teachers to explore misconceptions.

Card sorts, such as 'Equivalent Surds' (Activity B at the end of this article), were devised for this project and were given to teachers to try out on day 1 without any explanation about the design of them. It was absolutely fascinating to observe teachers' reactions to this particular activity. Upon realizing that there weren't just pairs of cards, some teachers laughed, some made remarks such as "Clever!" and others showed hints of frustration or mild anger that it wasn't what they had thought it would be. Many asked whether there were 'more than pairs', doubting their own ability or questioning whether there had been a mistake. These are probably the same reactions that might emerge in a classroom. This was often a moment of inspiration for teachers: having experienced it for themselves, they could see that it would be very easy to add challenge to a standard activity through making a small change to the resource.

One question that teachers asked was 'What do you want us to do with the blanks?' An obvious use for two of the blank cards is to write an expression to match each of the two that emerge as lone cards. What would you ask students to do with the other two? Some possible responses are: devise a new pair which utilizes $\sqrt{5}$, since there is only one set that currently includes it; make two of the pairs into threes (a bit harder); make a pair into a four (definitely harder); add a fifth card to the set of four (now you're being challenged); everyone in the class come up with a fourth one for each of the sets of three that you think no one else will come up with (that'll make them think!). As is often the case, devising the card set well requires more in-depth thinking than doing the task, so any opportunity for students to create instead of solve automatically adds to the challenge of the activity.

Even Better if...

Several of the resources were abject failures – or at least far less useful than had been hoped. Responding to feedback from teachers and students, activities such as 'Euclidean Algebra' and 'Fire and Rescue: Water Sources' were consigned to the dusty shelf. These were deemed 'too involved' and 'too far removed from the required curriculum' to be of real use to teachers, who often also said that they liked the activities, but couldn't see themselves using them with students.

Interestingly, the small numbers of students who did experience and review the Euclidean Algebra activity rated this as the activity which they enjoyed the most, which made them think most and which they felt had helped them make the most progress. This raises questions for professional development providers and resource developers about the extent to which they should persist in promoting activities which many teachers won't use, but the few that do will find them valuable.

Outcomes for Teachers

The feedback from teachers was illuminating, both in terms of the resources and the professional development sessions. Many teachers remarked that their usage of extension and enrichment materials had increased, with quotes such as "Improved! Because I've been a bit braver as I've had the opportunity to work through some of the problems before using them in the classroom", validating the design of the course. A large number of teachers reported that their frequency of usage of this type of material had remained the same. However, many qualified this with statements such as "The same, although I don't feel the same about how I use them! I am more confident that I pick the task more effectively and deliver the lesson more productively." Such comments, and the overall feedback, suggest that the strategy of using research to guide the design of both the resources and the course itself has proved successful.

What Next?

The two-day courses for academic year 2016/17 are already underway, but there will be a KS4 Extension and Enrichment conference in March and there are likely to be some one-day courses later in the year. The FMSP's web page www.furthermaths.org.uk/cpd has details of all the KS4 E&E and GCSE Higher courses.

References


Swan, M. 2005 Improving Learning in Mathematics, Department for Education and Skills Standards Unit.

Keywords: GCSE; Professional development; Extension and enrichment.

Authors MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.
**Activity A:**

**Algebraic Expressions 1 Maze**

Starting from the shaded cell, find a route to the opposite side of the rectangle so that each expression you land on is equivalent to \(a^2b\).

You may only move one space horizontally or vertically each time – no diagonal moves allowed!

<table>
<thead>
<tr>
<th>(2a^2b)</th>
<th>(a \times b^2)</th>
<th>(\frac{a^2b^3}{3})</th>
<th>(\frac{a^2b}{a})</th>
<th>(\frac{ab}{a^2b^2})</th>
<th>(2 \times a \times b)</th>
<th>(2b \times a)</th>
<th>(baa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a^2b)</td>
<td>(\frac{2ab}{\sqrt{a}})</td>
<td>(a \times a \times b)</td>
<td>(\frac{a^2}{b^2})</td>
<td>(\frac{(ab)^3}{ab^2})</td>
<td>((ab)a)</td>
<td>(\frac{(2ab)^2}{2b})</td>
<td>(\frac{a^2b^3}{b^2})</td>
</tr>
<tr>
<td>(\frac{a^3b^2}{a^b})</td>
<td>(\frac{(2ab)^2}{4b})</td>
<td>(2ab)</td>
<td>(ab \times ba)</td>
<td>(\frac{a^2b^3}{b^4})</td>
<td>(a \times 2 \times b)</td>
<td>(\frac{ab^2}{a^2b})</td>
<td></td>
</tr>
<tr>
<td>(a^2b)</td>
<td>((ab)^2)</td>
<td>(\left(\sqrt{a} \times b\right)^2)</td>
<td>(a + a + b)</td>
<td>(\frac{(2a^2b)^2}{4a^b})</td>
<td>(\frac{b}{a^2})</td>
<td>(\frac{(2a)^2 b}{2})</td>
<td>(\frac{a^3b}{a})</td>
</tr>
<tr>
<td>(2a \times b)</td>
<td>(\frac{(ab)^3}{ab})</td>
<td>(\frac{a}{(ab)^{-1}})</td>
<td>(\frac{a^3b^3}{a^3b^2})</td>
<td>(\frac{(ab)^2}{\sqrt{b}})</td>
<td>(\frac{a^5b^5}{a^3b^7})</td>
<td>(\sqrt{a^3b^2})</td>
<td>(\frac{a^4b^2}{2})</td>
</tr>
<tr>
<td>(a^2 \times b)</td>
<td>(ab \times ab)</td>
<td>((ba)^2)</td>
<td>(ab \times b)</td>
<td>(\frac{(2a \sqrt{b})^2}{2})</td>
<td>(\frac{a^5b}{a^3})</td>
<td>(\frac{2(ab)^2}{2b})</td>
<td>(\frac{(ab)^2}{b})</td>
</tr>
</tbody>
</table>

Mazes have been created for several topics, but teachers have also devised their own, and found that a great extension activity is to ask students to create one for themselves as this really challenges them to think about common errors.

**Activity B:**

**Equivalent Surds**

Cut out the cards (including the blanks) and match the equivalent ones.

<table>
<thead>
<tr>
<th>(2 + \sqrt{3})^2</th>
<th>11 + 6(\sqrt{2})</th>
<th>17 + 12(\sqrt{2})</th>
<th>12 – 6(\sqrt{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>6(2 – (\sqrt{3}))</td>
<td>7 + 4(\sqrt{3})</td>
<td>(3 – (\sqrt{3}))^2</td>
<td>(1 + (\sqrt{3}))(3 + (\sqrt{3})) + 1</td>
</tr>
<tr>
<td>(2 – 2(\sqrt{7}))^2</td>
<td>(3 + (\sqrt{2}))^2</td>
<td>(3 + 2(\sqrt{2}))^2</td>
<td>6(\sqrt{2}(1 + \sqrt{2})) – 1</td>
</tr>
<tr>
<td>32 – 8(\sqrt{7})</td>
<td>(3 + (\sqrt{7}))^2</td>
<td>14 + 6(\sqrt{5})</td>
<td>3 + 4(1 + (\sqrt{3}))</td>
</tr>
</tbody>
</table>
Introduction

Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. It is widely known as a leading curriculum development organization and has AS and A level Mathematics and Further Mathematics specifications examined through the OCR awarding body. MEI also provides support, through CPD and teaching and learning resources, for all AS and A level Mathematics specifications. This, the tenth article in the MEI Insights series, looks at MEI’s work to develop a Scheme of Work for the 2017 A levels.

Background

The new A level in mathematics for first teaching from September 2017 will inevitably present schools and teachers with both opportunities and challenges. In planning to deliver this new specification you may begin by asking what is changing. A key feature of the new A level is the requirement for students to demonstrate the knowledge and skills that are described in the following three Overarching Themes: mathematical argument, language and proof (OT1); mathematical problem solving (OT2); and mathematical modelling (OT3).

In addition the DfE guidance document states that ‘the use of technology, in particular mathematical and statistical graphing tools and spreadsheets, must permeate the study of AS and A level mathematics’ (DfE, 2014). These requirements, along with the move to a linear format in which the entire A level content will be examined at the end of a two-year course, will mean that a simple reshuffle of schemes of work developed for the current modular specification is unlikely to be sufficient.

In this article we will describe MEI’s approach to its freely available A level Scheme of Work (SoW) and offer suggestions for how you might use and adapt it in your department to suit your needs. Since the content of A level Mathematics is 100% prescribed by the DfE, the SoW will be suitable for use by all centres regardless of the awarding body that has been chosen.

Features of MEI’s Scheme of Work

The SoW is presented in 43 units, each focusing on a particular topic (see: www.mei.org.uk/2017-sow). The decision to do this was taken in recognition of the careful thought that had gone into designing the textbooks (see: www.hoddereducation.co.uk/Mathematics); each unit corresponds to a chapter in these textbooks. Whilst the order of the 43 units presents a coherent path through AS and A level Mathematics, it should be emphasized that teachers are not necessarily expected to use this order. For example, the units covering the mechanics topics in AS are presented together after all the AS pure units; this is to highlight that the mechanics ideas form a coherent body of mathematics in their own right. However, teachers are encouraged to think about teaching some mechanics alongside other topics such as calculus.

Each unit follows the same three-page structure:

- the content statements from the DfE document ‘Mathematics AS and A level content’ (DfE, 2014) along with a commentary on some points of conceptual or historical interest;
- two sample resources addressing the Overarching Themes and the effective use of technology;
- some suggestions related to the following issues that a department needs to consider: prerequisites for to see included?

by Bernard Murphy, Avril Steele and Stephen Lee
the unit, links with other topics, questions and prompts for mathematical thinking, opportunities for proof (in pure units) or modelling (in applied units), and common errors.

Some of these are illustrated below.

**Sample resource**

The aim is to provide a range of types of resource across the SoW with each one promoting one or more of the Overarching Themes. The resource on page 14 is taken from the A level 'Trigonometry' unit. Using the definition of radian measure (arc length divided by radius), students are challenged to draw appropriate diagrams to help them complete the grid. Is it necessary to provide the formulae for arc length or area of sector or might they work these out for themselves? Challenging students to tackle a task which is less structured and requiring them to justify and explain their placement of the numerical values supports elements of the Overarching Themes of mathematical argument, language and proof (OT1) and problem solving (OT2).

**Effective use of technology**

Across the SoW, a range of software, both teacher-led and student-led, is used, including Autograph, GeoGebra and spreadsheets, along with suggested uses of graphing calculators. At MEI, as stated in our MEI Insights 2 article on 'The use of technology in mathematics education': ‘we believe technology should be embedded within the teaching and learning of mathematics in a way that enhances students' mathematical learning by helping them to develop their understanding of mathematical concepts and by allowing them to access more mathematical ideas …’ The resource on page 14 is a student-centred use of graphing calculators taken from the AS level 'Polynomials’ unit. Through exploring numeric, graphic and algebraic features of a cubic function, students gain a deeper understanding of the factor theorem. For a free download of one year's subscription to the Casio fx-CG20 emulator visit [www.casio.co.uk/emulator/mei](http://www.casio.co.uk/emulator/mei) and for additional free resources visit [www.casio.co.uk/resources/mei](http://www.casio.co.uk/resources/mei).

**Questions and prompts for mathematical thinking**

The examples included in almost all of the units are inspired by the highly recommended ATM publication ‘Questions and Prompts for Mathematical Thinking’ (Watson and Mason, 1998). The following are taken from the AS level 'Surds and Indices' unit. How would your students respond? And how does the final question promote language, argument and proof (OT1)?

- Give me an example of a number that is equal to $3\sqrt{2}$ and another and a peculiar example.

- Change one digit in $\left(2 + \sqrt{8}\right)\left(\frac{4}{\sqrt{2}}\right)$ so that the product is a rational number.

- Give me an example of a number between $5\sqrt{6}+6\sqrt{5}$.

- $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. Always true, sometimes true, never true?

**Opportunities for proof**

Although one unit is devoted to proof, the SoW identifies opportunities to engage with proof in all the pure units. These are the suggestions from the A level ‘Sequences and Series’ unit.

- Prove that the infinite arithmetic sequence 3, 7, 11, 15, … contains no square numbers.

- Prove the formulae for the sum of arithmetic and geometric series.

- Prove that, for every triangular number $T$, $8T+1$ is a square number.

Can you find a geometric proof for the final one? (Nelsen, 1993). How would you use these alternative proofs with your students?

**Applications and modelling**

Mathematical modelling is the third Overarching Theme in the new A level. In several units of the SoW, suggestions are included for designing or commenting on a mathematical model. One example, from the A level ‘Differential Equations’ unit, takes the June 2012 OCR (MEI) Core 4 Comprehension paper (MEI, 2014) entitled 'The World’s Population', and explores the logistic equation, $\frac{dy}{dx} = ay(b-y)$ to model a scenario which has naturally occurring asymptotic behaviour.

**Adapting the Scheme of Work for your Department**

There is no single correct way to utilize the SoW. Indeed the intention is that it is flexible and can be adapted to suit your department’s needs. In tailoring it to fit your requirements a number of key questions are likely to arise and some key decisions will need to be taken. There will be organizational issues to address, such as whether the content will be delivered by a single teacher or by two (or more) teachers. How would you split the units from the SoW between teachers? If the option (or requirement) to sit the AS exam in Year 12 is to be retained for your students then some elements of both mechanics and statistics must be included in the first year of study. The total applied content remains at around a third so having a pure teacher and an applied teacher is not a good
match. Remembering that the Overarching Themes of problem solving and modelling must be addressed, could you embed mechanics and statistics with the pure maths content in order to facilitate this? Or would you prefer to deliver them in series and build more opportunity to apply pure maths in applied contexts as you go along?

No specific scheduling or time allocation is included in the SoW, so it has the freedom to be adapted to suit your timetable. How much time will you allocate to each unit? Which units with a common theme might you combine? Whilst the scheduling of units will clearly be designed to deliver the content in a logical order there are other factors that ought to be considered, making the exercise something of an iterative process. One factor might be the range of skills and expertise available in your department. Are all teachers prepared to teach the mechanics or statistics elements? Will all of your classes be able to access computing suites or other resources at appropriate stages to ensure that the use of technology permeates their studies? Another factor might be the impact of your choices on the delivery of Further Maths in parallel with Maths. The provision of pathways for Further Maths is likely be one of the initial structural decisions to be made, but having a good sense of how these might fit with your schemes of work could have a bearing on these decisions. Will your Further Maths students be in separate classes or integrated with Maths students? What would be the implications of teaching Further Maths in parallel with Maths? Or in series? Have you considered offering AS Further Maths to Year 13 students?

Teaching a linear course at A level will necessitate giving some thought to the way that revision is approached. Some units build upon and consolidate earlier units, but some key mathematical techniques, such as completing the square and the binomial expansion, may be covered in Year 12 and not necessarily revisited in Year 13. Will experience of building linear schemes for GCSE or planning for progression across Key Stages 3 and 4 be useful here? (See for example: www.ncetm.org.uk/resources/24350 – free login required to access) And what about the monitoring of progress and assessment? What are your departmental and school policies and how can these be catered for?

The MEI Scheme of Work as a Professional Development Tool

The final page in each unit is deliberately incomplete; it is designed for colleagues in departments to adapt together, sharing ideas and expertise. Below are two suggestions of how the SoW might be used in a departmental meeting.

Take any unit and spend time, first individually, then in pairs, then as a group, thinking of suggestions for each of the following:

- Prerequisites. What are the skills and knowledge you would expect students to bring to this topic?
- Links with other topics. Which topics overlap with this one? And what are the implications for teaching these units?
- Questions and prompts for mathematical thinking. Use a stem from the book (Watson and Mason, 1998) to design questions to promote mathematical thinking. For example, Change one aspect of ... so that ..., or Give me an example of ..., and another ..., and another.
- Opportunities for proof. Are there any proofs students can engage with in this (pure) topic? What would be more effective than simply showing them the proof?
- Applications and modelling. What opportunities are there in this topic to illustrate the modelling cycle? Can you think of a situation in which a likely initial model could be improved?
- Common errors. What mistakes do students often make? Why do they make them and what are the messages for your teaching?

Add your ideas to the editable SoW to start the process of making it your own.

An alternative use of departmental planning time might be to consider which resources can be adapted to fit another context. For example, how can a Venn Diagram, as used in the AS level ‘Equations and Inequalities’ unit, be used in other topics?

The inequality is satisfied by $x=2$
The solution has the form $a < x < b$ for integers $a$ and $b$
The inequality is satisfied by $x=4$

In Conclusion

The SoW developed by MEI is available at: www.mei.org.uk/2017-sow.

It provides resources, ideas and guidance for the 2017 mathematics AS and Alevels, which can be used, modified and developed by teachers to suit their own individual and departmental circumstances. We hope you find the SoW useful and we would be delighted to hear from teachers who are using it.


References

DfE 2014 GCE AS and A level Subject Content for Mathematics, DFE-00706-2014.


Keywords: Scheme of work; A levels; Maths; Further maths.

Authors MEI, Monkton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.

Arcs and Sectors

The numbers in the grid below are 12 of the 13 numbers in the grid on the right. What is the 13th missing number?

<table>
<thead>
<tr>
<th>$\theta$ (in radians)</th>
<th>$r$ (cm)</th>
<th>Arc Length (cm)</th>
<th>Perimeter of Sector (cm)</th>
<th>Area of Sector (cm$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td></td>
<td>20.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td></td>
<td></td>
<td></td>
<td>13.75</td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>


MEI Casio Tasks for AS Pure

Task 8: The Factor Theorem

1. Go into Table mode: [MENU] 7
2. Add Y1 = $x^3 - 2x^2 - x + 2$: $\boxed{x^3} - 2 x^2 - x + 2$
3. Use SET to set the table to Start: –5, End: 5, Step: 1: [F5] [F4] [5] [EXIT]
4. Display the table: [F6]
5. Go into Graph mode and plot the graph of this function: [MENU] 5 [F6]

Questions

- How do this table and graph confirm that $x^3 - 2x^2 - x + 2 = (x + 1)(x - 1)(x - 2)$?
- Can you find the factors of the following cubics:
  - $y = x^3 + 4x^2 + x - 6$
  - $y = x^3 - 4x^2 - 11x + 30$
  - $y = x^3 - x^2 - 8x + 12$
  - $y = x^3 - 7x^2 + 36$

Problem (Try the question with pen and paper first then check it on your calculator)

Show that $(x - 2)$ is a factor of $f(x) = x^3 + 4x^2 - 3x - 18$. Hence find all the factors of $f(x)$.

Further Tasks

- Find examples of cubics that only have one real root.
- Investigate using the factor theorem for polynomials of other degrees, e.g. quadratics or quartics.
- Investigate the polynomial solver: [MENU] [ALPHA] $\boxed{x,\beta,\gamma}$ [F2]

Introduction

Mathematics in Education and Industry (MEI) is an independent charity, committed to improving mathematics education. It works to support curriculum development and professional development for teachers, and to have a positive influence on national education policy. For a number of years MEI has offered professional development (PD) designed for teachers who have little or no experience of teaching the content of mechanics modules within AS/A level mathematics. In this article, the eleventh in the MEI Insights series, we outline our thoughts about what makes effective PD for teachers new to teaching mechanics, and in particular the challenges posed by the need to deepen specific subject knowledge and the pedagogical content knowledge required for mechanics to be taught effectively.

The Changing AS and A Levels in Mathematics

From September 2017 new AS/A levels in mathematics will be taught across the country. Among the changes is the requirement for all students to study mechanics and statistics alongside pure mathematics as part of both the AS and A level qualifications.

In light of these forthcoming changes we anticipate that the demand for PD in mechanics will increase as teachers will now be likely to be expected to teach mechanics having previously not done so. It is possible that many of these teachers will not have studied mechanics themselves. We describe some of the considerations we have taken into account when designing a PD course for teachers who are going to be new to teaching mechanics in 2017–19. These considerations are based on several years’ experience in designing and presenting mechanics PD for the current A levels.

We organize the one-day course around the two key themes of motion and force. Our aim is to place the teachers in the shoes of their students by considering the subject knowledge with which students arrive, demonstrate how to build upon that starting point, and illustrate some of the distinctive features of mechanics teaching through the activities we use. We outline some of these features below.

Practical Activities

One of the key features of effective mechanics teaching is the use of practical activities and experiments as a teaching device. Throughout the course we use short practical activities and demonstrations in order to draw out key principles and show how to develop the theory on the basis of what is observed.

Activity 1: The opening activity of the course day involves the teachers being given a displacement–time graph shown in Figure 1 and being asked to, ‘Walk out a journey which could be described by the graph.’

A simple activity like this underlines the fact that mechanics is an essentially practical subject which attempts to describe the real world. Teaching motion by asking students to move seems to make sense!

After participants have ‘walked out the journey’ we go on to explore whether the motion suggested by participants...
does result in the required graph. This is done in one of two ways. Either we use another practical activity where a toy car tracked by a sensor is used to replicate the motion, or we use a simple applet such as 'The Moving Man' (see screenshot in Figure 2 and Note 1) to generate the original motion graph. These both link the motion of an object to a motion graph and provide immediate feedback on the suggestions that have been put forward by participants.

Practical activities motivate interest among students and through the use of simple experiments students are able to 'test' a particular idea in order to see if they were right. This provides a useful vehicle for exploring misconceptions, as well as providing a focal point for classroom discussion.

**Misconceptions**

Maybe more than any other area of A level Mathematics, mechanics exposes misconceptions commonly held by students. The seminal publication, ‘Mechanics in Action’ (Savage and Williams, CUP, 1990) devotes a whole chapter to misconceptions (Note 2). One such scenario we use on the course is to ask teachers to observe a ball being thrown vertically into the air before considering an extract from 'Mechanics in Action' (Fig. 3). The fact that a number of participants perceive that the direction of the resultant force is determined by the direction of motion illustrates how deeply held misconceptions can be.

Related to the previous section, practical activities can be useful in highlighting common misconceptions held by students, as this example shows:

**Activity 2:** During a session about connected particles we hold up a ‘smooth’ pulley with two equal masses at differing heights connected by a light inextensible string and ask participants to predict what their students believe would happen when the string is released.

Responses include suggestions that the mass closer to the floor will move downwards ('because it’s already closer to the floor and therefore the pull of gravity is greater') and that the masses will move so as to be hanging at the

---

**Fig. 2**

![Graph showing motion parameters](image)

**Fig. 3**

![Diagram of a ball projected vertically](image)

---

2. A ball is projected vertically upwards. It rises through position $D$ until it reaches its highest point $E$ and then falls back down through $F$. Mark on the diagram an arrow which shows the direction of the resultant force acting at each position. Neglect air resistance.
same height (‘because that will make it fair and symmetry implies balance’). These sorts of responses provide a way of discussing what students really think is happening and exposing incorrect thinking.

One of the barriers teachers of mechanics have to overcome is the fact that our everyday language uses mechanics terminology in an informal sense. Students often have an understanding of concepts in A level mathematics such as weight, speeding up, slowing down, gravity, etc. which are drawn from them making sense of their own physical experiences. Drawing attention to this and equipping students with correct terminology while unpicking misconceptions is a skill developed by successful mechanics teachers.

Frequently, the work we do in highlighting and dealing with potential misconceptions is commented on by course participants. In answer to the question ‘What were the most useful aspects of the event?’ recent responses include:

‘Having my own misconceptions dealt with.’
‘Time to think like a student to address misconceptions.’
‘How to deal with misconceptions.’
‘Analysis of common misconceptions.’

These quotes illustrate the fact that misconceptions are an issue for some participants on the course. However, they also show that providing ideas for strategies for how student misconceptions can be exposed, discussed and corrected is valuable for these teachers.

**Modelling**

Mechanics is an attempt to model the real world using mathematics in order that we can then use the model to solve problems. The responses participants give in relation to Activity 2 (described above) naturally lead on to other areas of discussion, including that of developing a model. Use of the words light, inextensible and smooth routinely feature in A level mechanics questions and indicate some of the simplifying assumptions used when modelling a situation. Considering what would happen if these simplifying assumptions were not applied allows students to appreciate the importance of the simplifying assumptions within the model and the significance of such terms. Asking questions such as: What happens if the pulley is not ‘smooth’ and what effect, if any, would this have on the motion? How can we be sure that the tension in both parts of the string is the same? What happens if the string is not ‘light’? Can you justify why it is acceptable to ignore air resistance? allows students to develop a deeper understanding of the modelling process.

One of the ‘Overarching Themes’ as described in the DfE subject content document (Note 3) is ‘Mathematical Modelling’ which has particular relevance to the study of mechanics. The modelling cycle is something both teachers and students need to be familiar with and we anticipate that there will be an increased focus on this compared to current specifications. Students therefore need to be comfortable with how to develop and use a mathematical model. Mechanics provides a useful vehicle for this.

**Links Between Pure Mathematics and Mechanics**

One of the most significant changes in the structure of the new A level mathematics is that it will be a linear course. Given that pure and applied topics will form a single course rather than separate modules, the links between different areas of mathematics can be exploited to help provide a richer, more cohesive experience for students. Obviously, this approach has always been an option for teachers. However, for those new to teaching mechanics one of the challenges is seeing how one area of mathematics relates to another. We therefore felt it worthwhile to give time on the course to thinking about some of the links between pure and mechanics topics and how this content could be taught. We use a couple of different activities to help illustrate this:

**Activity 3:** Figure 4 shows two questions which are produced on small cards and handed to individuals. People seated next to each other have different questions. Each teacher is instructed to work on the question they have been given. Once complete, they and the person next to them are asked to compare their questions and approaches.

A particle is travelling in a straight line with constant acceleration. After 1 second its velocity is \(7 \text{ ms}^{-1}\) and after 2 seconds its velocity is \(11 \text{ ms}^{-1}\).

Find the initial speed and constant acceleration.

A line passes through the points A (1, 7) and B (2, 11).

Find the equation of the line and state the gradient and \(y\)-intercept.

Fig. 4
‘Making links between pure maths and its applications.’
‘How we can bring in calculus and how it relates to mechanics.’

We feel that by linking less familiar mechanics content to more familiar pure content has helped to deepen the teachers’ understanding. This more connected view of mathematics helps to build confidence in the participants’ subject knowledge and illustrates some of the underpinning principles in both mechanics and pure topics.

So What Happens Next?

Clearly, preparing teachers to teach content they haven’t taught before is a complex task and not something that can be achieved with a one-day course. We conclude the day by asking teachers to reflect on the main messages they will be taking away with them and encourage them to discuss with colleagues any questions that have been raised.

We therefore conclude this article by posing a few questions you might want to discuss in your department in order to review the mechanics teaching in the light of the forthcoming AS/A level changes:

Activity 4: This activity is a longer card-sorting activity. Designed to be used with students, it consists of nine cards showing pictures of graphs (Fig. 5) which have to be placed correctly on a $3 \times 3$ grid such that each column reads correctly. The labels on each row are given as either ‘Displacement–time’, ‘Velocity–time’ and ‘Acceleration–time’ or as ‘function’, ‘gradient function’ and ‘gradient of the gradient function’ (Fig. 6).

One of the key points teachers make after having worked on these two tasks is the observation that they switch between treating the task as a ‘pure maths task’ and as a ‘mechanics task’. The implications for teaching are then drawn out in discussion. Students, too, will be expected to develop a set of mathematical problem-solving skills which can be applied in a number of different contexts and some students will find it easier to think in pure maths terms while others will thrive on applying the mathematics in a context.

Helping teachers see some of the connections between different areas of mathematics has proved valuable. Feedback from course participants in answer to the question includes comments such as the following:

‘Resources presented in a very refreshing way; especially how calculus and physics are linked. Same with linking motion graphs with pure maths.’

Fig. 5
Q: If we walked into a mechanics lesson in our department would we be able to tell it was a mechanics lesson rather than a pure mathematics lesson and why?

Q: Which of the emphases outlined above – practical activities, use of the modelling cycle, identification and use of student misconceptions – do we do well in our teaching of mechanics?

Q: What links between pure mathematics and mechanics do we feel are important to emphasize? What changes do we need to consider in order for these links to be made in a way that helps to create a more coherent picture of A level mathematics?

Q: Is every member of the A level teaching team ready for teaching the new specifications? Are there teachers new to teaching mechanics and, if so, what support is needed for them to become effective mechanics teachers?

Fig. 6

Notes and References


Keywords: Professional development; Mechanics; A level mathematics.

Authors MEI, Monckton House, Epsom Centre, White Horse Business Park, Trowbridge, Wiltshire BA14 0XG.