

MEI Review of Mathematics in Other Subjects at Level 3 May 2012

1. Background

This curriculum review is carried out as part of MEI's 'Integrating Mathematical Problem Solving' project, funded by the Clothworkers' Foundation. The project aims are as follows.

- Highlighting the importance of mathematics and statistics across the sixth form curriculum
- Enabling students to see how mathematics and statistics are used in context across a variety of subjects
- Improving the ability of students to apply mathematical and statistical techniques to solve problems or draw conclusions
- Helping students to develop important skills for higher education and employment
- Inspiring students to investigate further

Further details can be found at www.mei.org.uk/IMPS

Mathematics and statistics are used in the study of many subjects in Higher Education. Sixth form students are not always aware of the mathematical and statistical demands of the subjects they choose to study to degree level. ACME's Mathematical Needs Report (2011) found the following.

We estimate that of those entering higher education in any year, some 330,000 would benefit from recent experience of studying some mathematics (including statistics) at a level beyond GCSE, but fewer than 125,000 have done so.

The Institute of Physics report, 'Mind the Gap' (2011), reported that 16% of Physics undergraduates stated that their degree course contained more mathematics than they had expected.

The UK Centre for Bioscience report, 'A survey of the mathematics landscape within bioscience undergraduate and postgraduate UK higher education' (2011), included the following recommendations.

- *A lack of mathematics content in A level Biology means that students do not expect to encounter maths at undergraduate level. There needs to be a more significant mathematical component in A level biology and chemistry along with opportunities for collaboration between academic bioscientists who use quantitative approaches and secondary maths and science teachers.*
- *In the light of reports from the Nuffield Foundation and the Royal Society it is important to consider broadening post-16 education to ensure that students are encouraged and have the opportunity to study mathematics that is rigorous and delivered in a scientific context.*

The Royal Statistical Society and Actuarial Profession report, The Future of Statistics in our Schools and Colleges (2012), stated the following.

It is particularly important that those responsible for education policy at school and college level recognise the importance of statistics in most degree courses, in employment as well as in enabling people to be informed citizens who evaluate evidence when making decisions.

2. What qualifications can students take at Level 3?

Students in sixth forms, studying at level 3, are likely to take qualifications from the list below. The availability of qualifications to students depends on what is offered by the school or college they are studying in.

- A Level
- Applied A Level
- Advanced Diploma
- BTEC (Business and Technology Education Council) National
- OCR Nationals Level 3
- International Baccalaureate
- Pre-U

The latest available national data about the proportions of the cohort taking different qualifications are in the DfE Guide to Post-16 Contextual Value Added (CVA) 2007 which states the proportions of students on the different routes as follows.

- A/AS Level (71.9 %)
- AVCE* (Advanced Vocational Certificate in Education) (5.7 %)
- NVQ/VRQ** (National Vocational Qualification/Vocationally Related Qualification) (4.0 %)
- BTEC/OCR Nationals (17.8%)
- IB (0.5 %)

* now withdrawn, last awarded in 2007

** normally studied by people in related employment

The following table shows more recent estimates of numbers taking level 3 qualifications, taken from a variety of sources.

Qualification type	Estimated number of students (2011)
A level (includes Applied A Level)	310 000
Advanced Diploma	1200
BTEC National	78 000
OCR Nationals Level 3	No information
International Baccalaureate	5000
Pre-U	2200

Appendix 1 gives more detail of these figures and their sources.

Students may also take other qualifications to support their main study, for example Functional Skills, Key Skills or Free Standing Mathematics Qualifications. A full list of qualifications counting for university entrance can be found in the UCAS tariff at http://www.ucas.com/students/ucas_tariff/qualifications

Most students going on to Higher Education take A Levels. Consequently, much of the rest of this review considers A Level subjects. However, it is recognised that some students preparing for Higher Education take a mixture of A Levels and other courses such as BTEC Nationals, Pre U or OCR Nationals. Other students take the IB Diploma or an Advanced Diploma.

The qualifications which are most similar in scope to traditional academic A Levels are the IB Diploma and the Pre U. Pre U is a new qualification and so it is too early for there to be any research into how well it prepares students for further study. Appendix 2 contains some information about the structure of the IB Diploma and how UK universities perceive it.

3. Relevant Questions

The following key questions are relevant when considering what is needed to ensure the mathematical needs of different subjects are met.

1. What mathematics is currently specified within different subjects at A Level?
2. Is the mathematics included within different A Level subjects appropriate and sufficient to meet the mathematical needs of students going on to higher education and/or employment in the subject areas concerned?
3. Do the teaching, learning and assessment of mathematics included within different A Level subjects properly reflect the spirit of the mathematical criteria in these subjects' specifications?
4. What can be done to develop the level 3 mathematics curriculum, both within other subjects and within mathematics qualifications, to ensure young people are appropriately prepared for the mathematics they will meet in higher education and employment?
5. Which of the following approaches would best enable students to be prepared for the mathematics they will need to progress beyond A Level?
 - Encouraging more students take AS/A Level Mathematics
 - New mathematics courses for post 16 students.
 - Increasing the mathematical content of other A Levels.
 - Encouraging students to do Extended Project Qualifications which include mathematical or statistical problem solving

This review aims to address questions 1 and 2 above. The Integrating Mathematical Problem Solving project, as a whole, aims to address questions 4 and 5.

In April 2012, SCORE reported on the mathematics in biology, physics and chemistry A Level assessments <http://www.score-education.org/policy/qualifications-and-assessment/mathematics-in-science> and the Nuffield Foundation reported on the mathematics in business studies, computing, economics, geography, psychology and sociology A level assessments. This partly addresses question 3 above.

The SCORE report included the following comments.

For biology, chemistry and physics, the analysis showed that the mathematical requirements that were assessed concentrated on a small number of areas (e.g. numerical manipulation) while many other areas were assessed in a limited way, or not at all.

A perceived consequence, raised repeatedly by the science community in the online survey, is that if mathematical content areas are frequently not assessed then these areas will not be taught or practised in depth. If areas within the mathematical requirements are not taught or practised, it will limit students' access to the subject, their ability to understand scientific concepts and reduce their mathematical fluency.

The Nuffield report included the following comments.

The report shows that it is possible for students who are ostensibly following the same course of study to have widely different levels of exposure to quantitative approaches to their subject. Stakeholders should consider the implications of this, and its advantages and disadvantages.

In some subjects the mathematical content or approaches are beyond those currently covered in GCSE. Where this is the case, it needs to be made explicit so that subject teachers can provide appropriate support for students and/or students can consider taking relevant post-16 mathematics qualifications.

4. Mathematics within different A Level Subjects

A Level subjects that are offered by more than one awarding body have to conform to national criteria; these criteria are available on the Ofqual website. Appendix 3 is a survey of individual subjects, detailing the mathematics and statistics their A Level specifications must contain.

5. Is the mathematical content of other A Levels sufficient to prepare students for further study?

The Institute of Physics report, 'Mind the Gap' (2011) reported as follows.

A large proportion (92%) of academics felt that a lack of fluency in mathematics was an obstacle to students achieving their full potential in the long term, and more than four in five (85%) agreed that a lack of fluency affected their department's ability to deliver an optimal programme of study.

It should be noted that these findings referred to students who had both A Level Physics and A Level Mathematics. The report recommended changes to A Level Mathematics to ensure that students' contextual understanding was better and an increase in crossover between A Level Mathematics and A Level Physics to ensure that students understand the connectedness of these subjects.

The UK Centre for Bioscience report, 'A survey of the mathematics landscape within bioscience undergraduate and postgraduate UK higher education' (2011), reported as follows.

Students enter bioscience undergraduate degrees with a very wide variety of mathematics qualifications from A at A2 Maths to less than C at GCSE. This wide variation causes difficulty in designing appropriate courses.

University staff noted that students' fear of mathematics was a more serious problem than their lack of knowledge.

A "fear of maths" or "maths-phobia" was commonly reported (12 out of 37 respondents) and it was noted that mature students in particular are more likely to lack confidence.

*"But by far the biggest problem is the *fear* of maths. There is a culture amongst students, which is perhaps encouraged at school, in which it acceptable (almost fashionable) to treat maths as some kind of mystical dark art, sent to terrorise biologists. I am sure a more positive attitude would allow them to overcome most of the issues we encounter with the kind of basic maths we ask them to use/understand.*

"The key difficulty is not so much their lack of knowledge as their lack of confidence – an unwillingness to dig in and use number to solve problems and better understand biological systems."

Academic staff report that students often do not expect to need any maths within a biology degree and the requirement for mathematical skills comes as a surprise.

"However, there are a significant proportion of students attracted to Biology that are quite poor in their maths skills, having not done post-GCSE maths. The maths content of a Biology degree comes as quite a shock to these students. I believe there should be more maths in both GCSE and A level Biology to help secondary students understand that it is part of modern biology."

The RSA report, 'Solving the maths problem: international perspectives on mathematics education' (2011) cited an earlier report from the University of Edinburgh: Proposals to support and improve the teaching of quantitative research methods at undergraduate level in the UK.

Improving the mathematics curriculum and assessment could also drive up standards in English universities. Some universities do not advertise the level of maths needed to comfortably study particular subjects for fear of hindering applications. Furthermore, recent research suggests that universities are marginalising mathematical content in the delivery of degree courses because English students are not capable of studying it or sometimes because the limited mathematical facility of teachers renders it difficult for them to teach advanced mathematical content. (MacInnes, 2009).

Even if the mathematics specified within other A levels is appropriate, these reports suggest the mathematics which students learn pre-university often leaves them poorly prepared for the mathematical demands of degree courses in a wide range of subjects.

6. Areas for curriculum development

For students who do not choose to study AS/A Level Mathematics, the following approaches represent possible ways to improve their knowledge of the mathematics and statistics required to support further study and employment.

- A. New mathematics courses for post 16 students.
- B. Increasing the mathematical content of other A Levels.
- C. Encouraging students to do Extended Project Qualifications which include mathematical or statistical problem solving

These are considered further below.

A. New Mathematics Courses for post 16 students

There are two obvious possible approaches in developing mathematics courses for students who do not wish to study AS/A Level Mathematics as a subject in its own right, but who need to develop their mathematical and statistical understanding in order to be prepared for further study and employment in their chosen fields:

1. Courses tailored to the needs of specific kinds of student could be developed.
2. More general courses could be developed, suitable for a wide range of students who do not wish to specialise in mathematics.

Both these approaches have been tried; courses which are currently available are outlined below.

A1. Tailored mathematics courses

The Advanced Diploma in Engineering is a level 3 course, roughly equivalent in size to 3.5 A Levels. The Diploma includes the following components.

- 540 guided learning hours (glh) Principal Learning
- 360 glh Additional or Specialist Learning
- 120 glh Extended Project
- 60 glh Functional Skills and personal learning and thinking skills
- 10 days work experience

Statistics available from www.jcq.org.uk show that the total number of students completing the Advanced Diploma in Engineering is as follows.

Year	Total number of students
2010	146
2011	177

The principal learning for the Advanced Diploma in Engineering includes a 60 guided learning hour compulsory mathematics unit. This is equal in size to one third of an AS in Mathematics.

Students can choose to take additional mathematics as part of the Additional or Specialist Learning in the Diploma. One option is to take the Level 3 Certificate in Mathematics for Engineering offered by OCR. This takes 180 guided learning hours and has been designed to enable students without A Level Mathematics to progress to an Engineering Degree.

The Level 3 Certificate in Mathematics for Engineering is one of two Level 3 certificates offered by OCR.

Statistics from the OCR website show that the total number of entries for Level 3 Certificates is as follows.

Session	Total number of entries
June 2010	12
Jan 2011	0
June 2011	18

Although it is possible for students to take an A Level in Mathematics as part of the diploma, it is not known how many students have done this. It is clear that there have been very few entries for the tailored mathematics course. This may indicate that highly tailored courses that are not compulsory are unlikely to have much take-up.

A serious practical problem with having a variety of tailored post-16 mathematics courses is that many sixth forms have relatively small numbers of students* and so would find it difficult or impossible to offer a choice of mathematics courses to their students because of staffing and financial restrictions.

*In 2011 45% of schools and colleges offering A level Mathematics had fewer than 15 students taking it (DfE).

A2. General courses for students who do not wish to specialise in mathematics

AS and A Level Statistics qualifications can provide useful preparation for the large number of students whose further study and employment includes the use of statistics. However, only a relatively small number of students currently take these qualifications.

AS Statistics is offered by AQA and MEI and the full A Level is offered by AQA only. The table below shows the total number of entries.

Year	AS Statistics	A Level Statistics
2010	1304	817
2011	1500	898

AQA's Level 3 Free Standing Mathematics Qualifications (FSMQs) were developed to enable students who do not take AS/A Level Mathematics, but who are following a path that requires mathematics beyond GCSE level, to study some level 3 mathematics. They are designed to be accessible to students with a grade C or above at GCSE, and aim to support the mathematics needed in other disciplines. Each FSMQ is equal in size to one third of an AS Level and it is possible to combine them into AS 'Use of Mathematics'. An A Level 'Use of Mathematics' qualification is currently being piloted. The table below shows the numbers of students taking FSMQs in the Use of Mathematics suite.

AQA Level 3 FSMQs available to all centres, June 2011 entry	
FSMQ Algebraic and Graphical Techniques	850
FSMQ Modelling with Calculus	207
FSMQ Using and Applying Decision Maths	208
FSMQ Using and Applying Statistics	540
AS Use of Mathematics (based on FSMQs)	647
Pilot AQA Level 3 FSMQs, June 2011 entry	
FSMQ Calculus	523
FSMQ Data Analysis	1947
FSMQ Decision Mathematics	1693
FSMQ Dynamics	171
FSMQ Hypothesis Testing	110
FSMQ Maths Principles for Personal Finance	132
AS Use of Mathematics (based on FSMQs)	1927
A Level Use of Mathematics (based on FSMQs)	510

Compared to the number of students taking A Level Mathematics each year (82 995 in 2011) the numbers taking Statistics and Use of Mathematics are very small. However, making such courses more widely available could result in increased numbers of students taking mathematics post 16. The Use of Mathematics pilot qualifications (and associated FSMQs) came under the scope of the 'Evaluating Mathematics Pathways' project [<http://www.nottingham.ac.uk/emp/>]. Their May 2009 report stated the following.

“One success criteria for the Pathways projects would be a substantial increase in the number of students studying mathematics post-16. Our evaluation suggests that FSMQs and AS/A2 Use of Mathematics qualifications provide a means of achieving this.”

B. Increasing the mathematical content of other A Levels

It is a reasonable expectation of students, universities and employers that A Level courses should provide a suitable preparation for further study. For subjects where the mathematical and statistical needs are not extensive, it may be possible to include them within the A Level specification of the relevant subject. This has the advantage of encouraging the required methods to be taught in relevant contexts. For this strategy to be successful, the following conditions need to be satisfied.

- Teachers need suitable resources and guidance
- Professional development needs to be available for teachers to enable them to teach the relevant mathematics and statistics with confidence.
- Assessments must adequately reflect and assess the mathematical and statistical understanding that students should acquire for further study in the subject.

For subjects with extensive mathematical and statistical requirements, additional courses will be required. For these subjects, it is essential that students, teachers and parents, as well as universities and employers, know that additional mathematics qualifications are needed to enable further progression.

C. Encouraging Extended Project Qualifications (EPQs)

It is encouraging to see the increasing take up of the level 3 EPQ (in 2009, 5094 students took the level 3 EPQ; by 2011 this figure had risen to 24099 students [<http://www.icq.org.uk/>]). The EPQ seems an effective way to broaden and extend students' education and encourage the development of independent thinking and learning skills. It can also provide an important vehicle for students to demonstrate significant use of mathematical and/or statistical analysis in context. However, it would not be realistic or desirable to envisage a future where all students undertook a project involving mathematical/statistical analysis. Even if they did so, this would not ensure that they developed all the mathematical and statistical understanding necessary for further study. Consequently, other ways of enabling students to study more mathematics need to be adopted.

Although EPQs on their own are not the answer to ensuring that students are properly prepared for the mathematical demands of higher education and/or employment, it would be valuable to encourage the use of mathematical and statistical analysis within EPQs across a wide range of subject areas.

Appendix 1

How many students take the different types of level 3 qualifications?

There is no single source of data for the numbers of students taking different level 3 qualifications.. The table below summarises information taken from a variety of sources. Notes about the sources are given below the table.

Qualification Type	Estimated number of students	Source of data
A Level	311 000	2011 Royal Society Report, <i>Increasing the Size of the Pool</i>
Applied A Level		
Advanced Diploma	1172	June 2011 JCQ results
BTEC National	Up to 170 131	Edexcel results information 2011 – due to differing sizes of BTEC qualification, it is possible that students can do more than one
OCR Nationals Level 3	No information	
International Baccalaureate	5017	IBO Statistical Bulletin May 2011
Pre-U	2115	CIE press release 19 August 2010 for first year's results; September 2011 pre U Newsletter for percentage increase in entries.

The JCQ 2011 results release of 18 August stated that a total of 1 411 919 A Levels were completed in the UK in summer 2011, including those taken by mature learners. The majority of A level students take three or four A levels. In addition to these A Levels, there were 34 728 Single Award Applied A Levels and 7434 Double Award Applied A Levels. The 2011 Royal Society Report, 'Increasing the Size of the Pool', estimated the A Level cohort size in England, Wales and Northern Ireland as 311 000; this excludes mature students. This figure is consistent with 250 000 taking A Levels in England reported by Ofqual in their International Comparisons in Senior Secondary Assessment (2012).

The Joint Council for Qualifications (JCQ) provisional Advanced Diploma results for June 2011 show that 1172 Advanced Diplomas were awarded, across all subjects. An Advanced Diploma is roughly equal in size to three and a half A Levels so students will not take more than one Advanced Diploma.

Edexcel run BTEC qualifications. BTEC National qualifications are at Level 3; as vocational qualifications, they are available in different sizes. From September 2010, these are as follows.

- BTEC Level 3 Extended Diploma, equivalent in size to three A Levels
- BTEC Level 3 Diploma, equivalent in size to two A Levels
- BTEC Level 3 Subsidiary Diploma, equivalent in size to one A Level
- BTEC Level 3 Certificate, equivalent in size to one AS Level

Edexcel results data show that 121 131 students in 2011 completed a qualification equal in size to at least two A Levels; with a further 48 996 completing a smaller qualification. It is not known what proportion of these students are over the age of 19. The figure of 78 000 given in the earlier table in this review is based on the assumption that the ratio between A Level and BTEC students is consistent with that in 2007.

OCR Nationals at Level 3 are available in the following sizes.

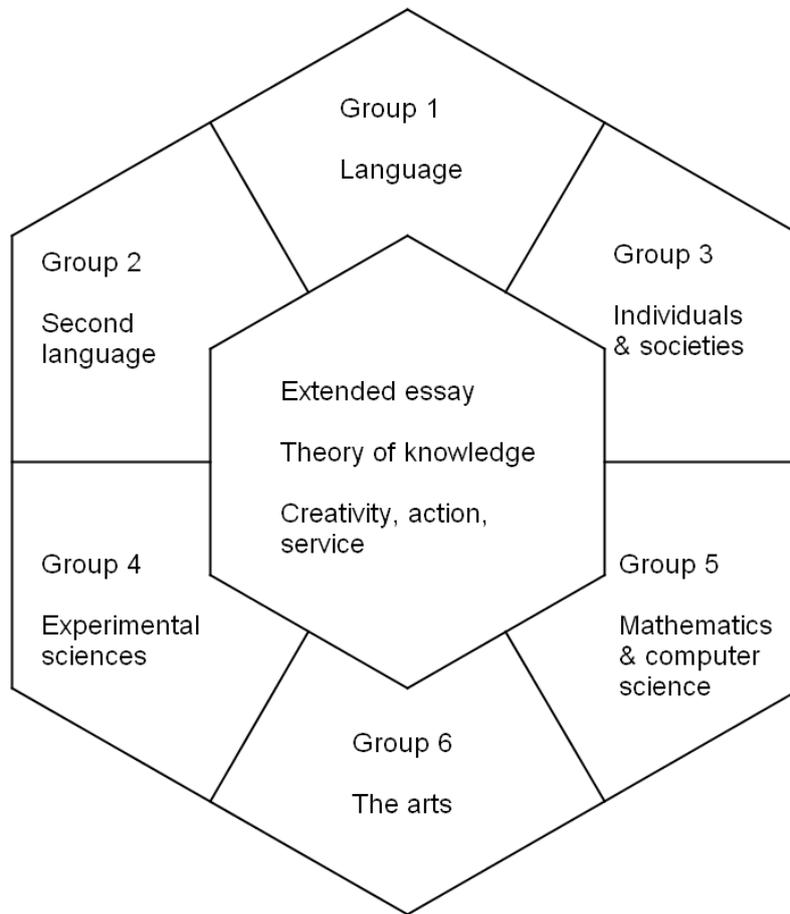
- Certificate, equivalent in size to one A Level.
- Diploma, equivalent in size to two A Levels.
- Extended Diploma, equivalent in size to three A Levels.

The IB Diploma is run by the International Baccalaureate Organisation and is done by candidates across the world. Students take a spread of subjects to make up the diploma. Further details are given in Appendix 2.

CIE Pre U examinations are similar in size to A Levels but Pre U are linear qualifications which have all the assessment at the end of the course. First results were in August 2010.

Appendix 2

The IB Diploma Programme



Candidates for the IB Diploma study one subject from each of groups 1 to 5, shown above. They also choose a sixth subject from either group 6 or an additional subject from groups 1 to 5. Each subject is studied at either standard level or higher level. Either three or four subjects are taken at higher level, with the others being taken at standard level.

Higher level courses are 240 teaching hours, whereas standard level courses are 150 hours. This compares to 360 guided learning hours for an A Level.

In addition to the six subjects, Diploma candidates study theory of knowledge as a unifying discipline, write an extended essay of around 4 000 words and take part in activities outside the classroom for the creativity, action and service requirement.

Perceptions of the International Baccalaureate Diploma Programme, Jenkins (2003) evaluated the IB Diploma as a suitable preparation for degree level study in the UK. This involved surveying 71 institutions with follow up interviews at 20 of these. 96% of respondents to the questionnaire were satisfied that IB Diploma students were adequately prepared for undergraduate study.

IB Diploma students study a broader range of subjects than A Level students but spend less time on each. Specific comments from Jenkins (2003) which are relevant to this review are reproduced below.

Respondents at The Universities of Kent and Brighton speculated about mathematics and physics but respondents from Kent stressed that “IB students tended to do better”.

London School of Economics and Political Science felt that there was a stark contrast between DP and A-level students with the former at an advantage, and that the compulsory inclusion of mathematics was a distinct advantage.

Imperial College of Science, Technology and Medicine was satisfied with DP mathematics and commented on the constancy of standards and curriculum over a period. The DP mathematics examinations were seen to be more testing of a student’s mathematical training than A-level examinations.

At University of Bath the mathematics department is satisfied with the preparation of DP students and felt that they bring good communication skills. Higher or standard level DP mathematics is much valued in the departments of economics, international development, and business and management.

Appendix 3

A survey of the Mathematics specified in the A level criteria for different subjects

Extracts from the national criteria are used under the terms of the Open Government Licence and are shown below in italics.

It should be noted that some criteria list all mathematical topics in detail whereas others give briefer outlines; the number of words used does not always indicate the importance of mathematics and statistics in the assessment or the teaching of the subject.

Business Studies

The A Level criteria for Business Studies account for 60-100% of the content. The aims for the subject include encouraging learners to do the following.

acquire a range of relevant business and generic skills, including decision making, problem solving, the challenging of assumptions and the quantification and management of information.

The AS core includes “Accounting and finance” which comprises the following aspects.

- *Budgeting.*
- *Cash-flow forecasting.*
- *Break-even analysis.*

The A2 core includes the following topics.

- *Risk and uncertainty.*
- *Forecasting.*
 - *Data analysis.*
 - *Market analysis.*
 - *Decision making.*
 - *Measures of performance: financial and non-financial*

Applied Business

Applied Business is available as either a single or a double award. The latter contains more content devised by individual awarding bodies. The criteria give the following guidelines.

In addition to the areas of study in the core content, specifications must include other areas of study related to business in vocational settings, to make up the remainder of the following specifications:

- *AS (three-unit): no awarding organisation devised content;*
- *AS double award (six-unit): one-half awarding organisation devised content;*
- *A level (six-unit): one-third awarding organisation devised content;*
- *A level double award (12-unit): two-thirds awarding organisation devised content.*

The aims and objectives include the following statement.

All specifications should also encourage learners to:

- *apply numerical and written business techniques to a variety of business contexts;*
- *explore business problems and learn to identify possible solutions*

The AS core content includes the following.

- *Investigating business*
 - *business planning;*
 - *monitoring and reviewing business activities;*
 - *resource management and quality control.*
- *Introduction to marketing*
 - *market research;*
- *Financial management*

The emphasis should be on management accounting to aid decision making, including:

 - *cash-flow management;*
 - *the use of software, for example spreadsheets, for record-keeping and other financial purposes;*
 - *profit and break-even analysis;*
 - *setting and monitoring budgets.*

The A Level core content includes the following.

- *Business investigation*

The investigation should encompass:

 - *aims and objectives;*
 - *target market / market research;*
 - *marketing issues;*
 - *financial analysis and planning;*
 - *resource management;*
 - *recommendations;*
 - *evaluation of outcomes and/or processes.*

Economics

The A Level criteria for Economics account for 60-100% of the content. Economics A Level specifications must put emphasis on the use of data and models, including requiring students to do the following.

- *select, interpret and use appropriate data from a range of sources;*
- *develop a critical approach to economic models and methods of enquiry.*

At AS Level, learners must be required to:

- *understand simple micro-economic and macro-economic market models with a limited number of variables; use the models to explore current economic behaviour; make causal connections; and develop an understanding of how the models shed light on the economy as a whole;*
- *be aware of the assumptions of the basic model of supply and demand; explain the way it works in both words and diagrams; and use the model to describe, predict and analyse economic behaviour;*
- *use the basic aggregate supply/aggregate demand (AS/AD) model and data to understand why supply-side and/or demand-side policies may be seen as appropriate ways of managing an economy; predict the possible impact of such policies and recognise the assumptions involved; argue for different approaches; and identify criteria for success.*

In addition, at A Level learners must:

- *use and evaluate more complex models involving more variables;*
- *apply models to a wider range of contexts;*
- *develop the ability to apply and evaluate economic models as represented in written, numerical and graphical forms;*
- *interpret and evaluate different types of data from multiple sources;*
- *be able to propose possible solutions to problems;*
- *understand the relationships and linkages that underpin macro-economic models;*

Specific content which includes mathematics is as follows.

- *Allocation of resources.*
- *Supply and demand.*
- *Elasticity.*
- *Economic policy objectives and indicators of macro-economic performance. For example economic growth, employment, inflation, the balance of payments, income distribution and welfare.*
- *Exchange-rate changes.*

Geography

The Geography A Level criteria are stated in terms of what learners should be required to do. This includes the following.

AS

- *use a range of skills and techniques, including the use of maps and images at different scales necessary for geographical study;*
- *carry out research, and out-of-classroom work including fieldwork, as appropriate to the topics selected*
- *use modern information technologies, including geographical information systems, as appropriate to the content;*

A2

- *undertake individual research/investigative work, including fieldwork*
- *analyse and synthesise geographical information in a variety of forms and from a range of sources;*
- *critically reflect on and evaluate the potential and limitations of approaches and methods used both in and outside the classroom.*

The brevity of the Geography A Level criteria does not convey the extent of statistical investigation incorporated into A Level specifications in Geography. Looking at the specifications from the English awarding bodies makes it clear that statistical work forms a significant part of the assessment of the subject. For example, the OCR specification has “Geographical skills” as one of the four units in the specification; this unit includes the following skills.

“ Identifying a suitable geographical question or hypothesis for investigation
Developing a plan and strategy for conducting the investigation
Collecting and recording appropriate data
Presenting the data collected in appropriate forms
Analysing and interpreting the data”

The recent report “The Future of Statistics in our Schools and Colleges” identifies the following statistical hypothesis tests as being used in A Level Geography.

- Spearman’s rank correlation
- X^2 test
- Mann-Whitney U test

Science

A Level Science criteria cover the following subjects.

- Biology
- Chemistry
- Physics
- Psychology
- Geology
- Electronics
- Environmental Science

The criteria include requirements to include “How science works” in all subject specifications. This includes the following aspects.

- *Carry out experimental and investigative activities, including appropriate risk management, in a range of contexts.*
- *Analyse and interpret data to provide evidence, recognising correlations and causal relationships.*
- *Evaluate methodology, evidence and data, and resolve conflicting evidence.*

The criteria include a list of mathematical content for science subjects along with a table showing which subject(s) each part of this content is used in. This is reproduced in full below.

Mathematical content for science subjects

89. In order to be able to develop their skills, knowledge and understanding in science, learners need to have been taught, and to have acquired competence in, the appropriate areas of mathematics relevant to the subject as indicated in the table of coverage below.

90. Arithmetic and numerical computation

90.1 Recognise and use expressions in decimal and standard form.

90.2 Use ratios, fractions and percentages.

90.3 Make estimates of the results of calculations (without using a calculator).

90.4 Use calculators to find and use power, exponential and logarithmic functions.

90.5 Use calculators to handle $\sin x$, $\cos x$, $\tan x$ when x is expressed in degrees or radians.

90.6 Use hexadecimal and binary systems.

91. Handling data

91.1 Use an appropriate number of significant figures.

91.2 Find arithmetic means.

91.3 Construct and interpret frequency tables and diagrams, bar charts and histograms.

91.4 Understand simple probability.

91.5 Understand the principles of sampling as applied to scientific data.

91.6 Understand the terms mean, median and mode.

91.7 Use a scatter diagram to identify a correlation between two variables.

91.8 Use a simple statistical test.

91.9 Make order of magnitude calculations.

92. Algebra

- 92.1 Understand and use the symbols: =, <, <<, >>, >, α , \sim .
- 92.2 Change the subject of an equation.
- 92.3 Substitute numerical values into algebraic equations using appropriate units for physical quantities.
- 92.4 Solve simple algebraic equations.
- 92.5 Use logarithms in relation to quantities that range over several orders of magnitude.

93. Graphs

- 93.1 Translate information between graphical, numerical and algebraic forms.
- 93.2 Plot two variables from experimental or other data.
- 93.3 Understand that $y = mx + c$ represents a linear relationship.
- 93.4 Determine the slope and intercept of a linear graph.
- 93.5 Calculate rate of change from a graph showing a linear relationship.
- 93.6 Draw and use the slope of a tangent to a curve as a measure of rate of change.
- 93.7 Understand the possible physical significance of the area between a curve and the x axis and be able to calculate it or measure it by counting squares as appropriate.
- 93.8 Use logarithmic plots to test exponential and power law variations.
- 93.9 Sketch simple functions including $y = k/x$, $y = kx^2$, $y = k/x^2$, $y = \sin x$, $y = \cos x$, $y = e^{-x}$.

94. Geometry and trigonometry

- 94.1 Appreciate angles and shapes in regular 2D and 3D structures.
- 94.2 Visualise and represent 2D and 3D forms including two-dimensional representations of 3D objects.
- 94.3 Understand the symmetry of 2D and 3D shapes.
- 94.4 Calculate areas of triangles, circumferences and areas of circles, surface areas and volumes of rectangular blocks, cylinders and spheres.
- 94.5 Use Pythagoras' theorem, and the angle sum of a triangle.
- 94.6 Use \sin , \cos and \tan in physical problems.
- 94.7 Understand the relationship between degrees and radians and translate from one to the other.

Table of coverage

	Biology	Chemistry	Physics	Psychology	Geology	Electronics	Environmental Science
90.1	✓	✓	✓	✓	✓	✓	✓
90.2	✓	✓	✓	✓	✓	✓	✓
90.3	✓	✓		✓	✓		✓
90.4	✓	✓	✓			✓	✓
90.5			✓			✓	
90.6						✓	
91.1	✓	✓	✓	✓	✓		✓
91.2	✓	✓	✓	✓	✓		✓
91.3	✓			✓	✓		✓
91.4	✓			✓	✓		✓
91.5	✓			✓	✓		✓
91.6	✓			✓	✓		✓
91.7	✓			✓	✓		✓
91.8	✓			✓	✓		✓
91.9			✓	✓	✓		✓
92.1		✓	✓	✓	✓	✓	✓
92.2	✓	✓	✓			✓	
92.3	✓	✓	✓	✓	✓	✓	✓
92.4		✓	✓			✓	
92.5		✓					
93.1	✓	✓	✓	✓	✓	✓	✓
93.2	✓	✓	✓	✓	✓	✓	✓
93.3		✓	✓		✓		✓
93.4		✓	✓		✓	✓	
93.5	✓	✓			✓		
93.6		✓	✓			✓	
93.7			✓				
93.8			✓				
93.9			✓				
94.1		✓					
94.2		✓			✓		✓
94.3		✓					
94.4			✓		✓		✓
94.5			✓				
94.6			✓				
94.7			✓				

In addition to these general requirements, there are specific requirements for each subject which are outlined below.

Biology

The A Level criteria for biology account for 60% of the total content. Areas of compulsory content which involve mathematical aspects are listed below.

- *The dynamic equilibrium of populations is affected by a range of factors.*
- *Negative feedback helps maintain an optimal internal state in the context of a dynamic equilibrium. Positive feedback also occurs.*

The recent report “The Future of Statistics in our Schools and Colleges” identifies Biology as a subject where students are likely to use the whole statistics cycle consisting of the following stages.

- Problem analysis
- Data collection
- Data presentation
- Data analysis

The report also highlights the following statistical hypothesis as being likely to be used in A Level Biology.

- Spearman’s rank correlation
- X^2 test
- t- tests

Chemistry

The A Level criteria for chemistry account for 60% of the total content. Areas of compulsory content which involve mathematical aspects are listed below.

Formulae, equations and amounts of substance

- *Empirical and molecular formulae.*
- *Balanced chemical equations (full and ionic).*
- *The Avogadro constant and the amount of substance (mole).*
- *Relative atomic mass and relative isotopic mass.*
- *Calculation of reacting masses, mole concentrations, volumes of gases, per cent yields and atom economies.*
- *Simple acid–base titrations.*
- *Non-structured titration calculations, based solely on experimental results.*

Energetics

- *Enthalpy changes, including standard enthalpy changes of reaction, formation and combustion. Average bond enthalpies.*
- *Use of Hess's law to calculate enthalpy changes.*

Kinetics

- *A qualitative understanding of collision theory. Activation energy and its relationship to the qualitative effect of temperature changes on rate of reaction.*
- *Determination and use of rate equations of the form: $\text{Rate} = k[\text{A}]^m[\text{B}]^n$, where m and n are integers. Using orders of reactions where appropriate, which may give information about a rate-determining/limiting step.*

Equilibria

- *The dynamic nature of equilibria. For homogeneous reactions, the qualitative effects of temperature, pressure and concentration changes on the position of equilibrium. Equilibrium constants, K_c . Calculation of K_c and reacting quantities.*
- *The Bronsted–Lowry theory of acid–base reactions. The ionic product of water, K_w ; pH and its calculation for strong acids and strong bases.*
- *Dissociation constants of weak acids, K_a . Calculation of pH for weak acids. Buffer solutions and their applications*

Physics

The A Level criteria for physics account for approximately 60% of the total content. Areas of compulsory content which involve mathematical aspects are listed below.

All Physics specifications should require learners to develop:

- *their knowledge of SI units;*
- *an understanding of the distinction between vector and scalar quantities;*
- *an awareness of the order of magnitude of physical quantities;*
- *an awareness of the limitations of physical measurements.*

All Physics specifications must ensure that there is an appropriate balance between mathematical calculations and written explanations of principles.

Mechanics

- **Vectors:**
 - *resolution into two components at right angles*
 - *addition rule for two vectors*
 - *calculations limited to two perpendicular vectors.*
- **Kinematics:**
 - *graphical representation of uniform accelerated motion*
 - *use of kinematic equations in one dimension with constant velocity or acceleration*
 - *interpretation of speed and displacement graphs for motion.*
- **Dynamics**
 - *use of $F = ma$ when mass is constant;*
 - *one- and two-dimensional motion under constant force;*
 - *independent effect of perpendicular components with non-uniform acceleration.*
- *Energy calculation of work done for constant forces, including force not along the line of motion calculation of exchanges between gravitational potential energy and kinetic energy.*
- **Momentum:**
 - *definition, equation;*
 - *principle of conservation of momentum;*
 - *calculations for one-dimensional problems*
- **Circular motion:**
 - *application of $F = ma = mv^2/r$ to motion in a circle at constant speed.*
- **Oscillations:**
 - *simple harmonic motion;*
 - *quantitative treatment, limited to $a = -(2\pi f)^2x$ and the solution $x = A \cos 2\pi ft$;*
 - *velocity as gradient of displacement–time graph;*
 - *qualitative treatment of free and forced vibrations;*
 - *damping and resonance*

Electric circuits

- **Current:**
 - electric current as rate of flow of charge, $I = \Delta q / \Delta t$.
- **Circuits:**
 - conservation of charge and energy in simple circuits;
 - relationships between currents, voltages and resistances in series and parallel circuits;
- **Resistance:**
 - definition;
 - resistivity;
 - Ohm's law as a special case;
 - power dissipated.
- **Capacitance:**
 - definition;
 - energy of a capacitor;
 - quantitative treatment of discharge curves.
- **Waves:**
 - path difference, phase and coherence;
 - graphical treatment of superposition and standing waves

Matter

- **Molecular kinetic theory:**
 - ideal gases; $pV = NkT$;
 - absolute zero;
 - effect of temperature on average molecular kinetic energy;
 - energy of an ideal gas.
- **Internal energy:**
 - energy required for temperature change = $mc\Delta\theta$.

Quantum and nuclear physics

- **Nuclear decay:**
 - modelling with constant decay probability leading to exponential decay; idea of half life;
- **Nuclear energy:**
 - $E = mc^2$ applied to nuclear processes;
 - appreciation that $E = mc^2$ applies to all energy changes;
 - simple calculations relating mass difference to energy change;

Fields

- **Force fields c:**
 - concept and definition;
 - gravitational force and field for point (or spherical) masses;
 - electric force and field for point (or spherical) charges in a vacuum ;
 - uniform electric field;
 - similarities and differences between electric and gravitational fields.
- **B-fields:**
 - force on a straight wire and force on a moving charge in a uniform field with field perpendicular to current or motion.
- **Flux and electromagnetic induction:**
 - Faraday's and Lenz's laws;
 - emf as equal to rate of change of magnetic flux and simple calculations.

Psychology

The A Level criteria for psychology account for approximately 60% of the total content. Areas of compulsory content which involve mathematical aspects are listed below.

AS level specifications must also require learners to develop knowledge and understanding of research methods in psychology including:

- *methods and techniques for collection of quantitative and qualitative data including experimentation, observation, self-report and correlation;*
- *experimental design including independent measures and repeated measures;*
- *descriptive statistics including measures of central tendency dispersion and graphical presentation of results*

In addition to the AS level requirements, A level specifications must require learners to further develop knowledge and understanding from at least two of the core areas Knowledge and understanding must be related to:

- *the selection and application of knowledge and understanding of theories, concepts and approaches to the solution of problems;*
- *the design and reporting of investigations and drawing valid conclusions from them;*
- *the collection and analysis of both quantitative and qualitative data including the use of inferential statistics;*

The recent report “The Future of Statistics in our Schools and Colleges” identifies Psychology as a subject where students are likely to use the whole statistics cycle consisting of the following stages.

- Problem analysis
- Data collection
- Data presentation
- Data analysis

The report also highlights the following statistical hypothesis as being likely to be used in A Level Psychology.

- Spearman’s rank correlation
- Product moment correlation
- X^2 test
- Mann-Whitney U test
- Wilcoxon signed rank test
- Sign test
- t- tests

Geology

The A Level criteria for geology account for approximately 60% of the total content. Areas of compulsory content which involve mathematical aspects are listed below.

Geological time

- *Principles of dating.*

Geological data

- *Collection and interpretation of geological data including maps and photos, logs and other data.*

Climate change

- *Evidence and impacts over varying timescales, past climates and their interpretation in the rock record.*

The specifications for A Level Geology include fieldwork where candidates collect and work with data.

Environmental Science

The A Level criteria for environmental science account for approximately 60% of the total content. Areas of compulsory content which involve mathematical aspects are listed below.

Learners must investigate the evidence for human impact on the environment (both positive and negative), environmental problems, solutions and their validity to enable informed discussion and decision making.

Electronics

The A Level criteria for electronics account for approximately 60% of the total content. Areas of compulsory content which involve mathematical aspects are listed below.

System synthesis

- Recognise that simple systems consist of an input, a process, an output and possibly feedback analyse and design system diagrams.
- Represent complex systems in terms of sub-systems.

Logic systems

- Identify and use NOT, AND, NAND, OR, NOR and EOR gates.
- Construct and recognise truth tables for these gates and simple combinations of them use combinations of one type of gate to perform other logic functions.
- Generate the Boolean expression for a system from a truth table.
- Simplify a logic system.

Voltage (V), current (I)

- Define resistance as V/I and resistance (R).
- Calculate the combined resistance of resistors connected in series and/or parallel.

Power

- Define power as VI .
- Apply the formula to calculate power dissipation in a circuit.

Resistive input transducers

- Describe the use of LDRs, negative temperature coefficient thermistors and switches in a voltage divider circuit to provide analogue signals.
- Interpret and use the characteristic curves of the above devices.

Op-amps

- Use the formula: $\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$
- Use the formula: $\frac{V_{\text{out}}}{V_{\text{in}}} = 1 + \frac{R_f}{R_1}$

Timing circuits

- Calculate the value of the time constant for RC circuits.

Sequential logic sub-systems

- Construct and use timing diagrams to explain the operation of sequential logic circuits recall and describe the operation of a binary ripple up-counter.

Radio reception .

- Calculate the resonant frequency of a tuned circuit.

Applied Science

Applied Science is available as either a single or a double award.

The AS core content includes the following.

- *Data handling*
 - *Choosing and evaluating sources of data*
 - *Obtaining and using primary and secondary data*
 - *Uses and limitations of qualitative and quantitative data*
 - *Interpretation, explanation and evaluation of data*

- *Energy*
 - *Energy changes in chemical reactions and relationship to bond making and breaking*
 - *Application to respiration and burning fuels*
 - *Applications of energy transfer*
 - *Controlling temperature and rate of energy transfer in a system*
 - *Consequences of wasteful energy transfer and methods of reducing this*
 - *Limits to the efficiency of energy transfer*

The A2 core content includes the following.

- *Planning an investigation*

Production of a plan for an investigation related to a vocational context, which includes:

 - *identification of sources of information and awareness of the need to check for validity;*
 - *selection of appropriate techniques and equipment;*
 - *identification of constraints, including health and safety regulations and risk assessments.*

- *Carrying out the investigation*
 - *Implementation and review of the planning*
 - *Collection of reliable quantitative and/or qualitative data.*
 - *Recording of data, if numerical, to an appropriate degree of precision*
 - *Methods of checking the reliability of the data*

- *Processing and presenting data in investigation*
 - *Use of appropriate methods to process the data*
 - *Treatment of any anomalous data collected*
 - *Presentation of results in a suitable format*

- *Evaluation of the investigation*
 - *Interpretation of results and drawing conclusions*
 - *Evaluation of methods used and, if appropriate, suggestions for improvement*
 - *Discussion of the significance of conclusions*
 - *Production of a report suitable for the identified audience. This could be a written report, presentation, poster, video or any other appropriate medium*

Applied Engineering

Applied Engineering is available as either a single or a double award. The latter contains more content devised by individual awarding bodies.

The AS core includes the following content.

- *Designing and project management*
Understand and apply the knowledge and skills applicable to the design process within the context of project-based work, including:
 - *understanding client briefs;*
 - *planning a project, including planning, prototype and manufacture;*
 - *analysing and evaluating a design solution or project outcome;*
 - *project presentation and report writing;*
 - *preparing a design solution;*
 - *project management.*

The A Level core includes the following content.

- *Characteristics, applications and properties of materials*
Select materials for engineering applications according to their properties, including:
 - *performing simple calculations of stresses on simple structures, including pin-jointed frames and sections.*
- *Electronics, instrumentation and control*
Investigate the application of electronics in a range of integrated engineering systems in one or more of the following contexts:
 - *control;*
 - *monitoring;*
 - *measurement;*
 - *communication.*
- *Designing and project management*
Understand and apply the knowledge and skills applicable to the design process within the context of project-based work, including:
 - *understanding client briefs;*
 - *planning a project, including planning, prototype and manufacture;*
 - *analysing and evaluating a design solution or project outcome;*
 - *project presentation and report writing;*
 - *design for cost, manufacture and sustainability;*
 - *preparing a design solution;*
 - *project management.*
- *Application of mathematics and science*
Apply mathematical and scientific principles to solve engineering problems.

Sociology

The statistical aspects of A Level sociology specifications are outlined below.

GCE AS and A level specifications will require learners to demonstrate knowledge and understanding of a range of methods and sources of data and to understand the relationship between theory and methods, particularly the way sociologists deal with:

- *the collection of primary and secondary data;*
- *the analysis of quantitative and qualitative data using appropriate concepts;*
- *factors influencing the design and conduct of sociological research;*
- *practical, ethical and theoretical issues arising in sociological research.*

Collection and recording of evidence*

GCE AS and A level specifications will require learners to demonstrate their ability to:

- *analyse and evaluate the design of sociological investigations;*
- *analyse and evaluate the method(s) used in these investigations to collect and record evidence.*

This could be achieved by learners designing and conducting a sociological investigation.

Interpretation and evaluation of evidence*

GCE AS and A level specifications will require learners to demonstrate their ability to:

- *distinguish between facts, opinions and value judgements;*
- *select and apply a range of relevant concepts and theories;*
- *interpret qualitative and quantitative data;*
- *identify and evaluate significant social trends shown in evidence;*
- *evaluate theories, arguments and evidence.*

** The term evidence should be understood to include both primary and secondary sources, as well as both quantitative and qualitative data.*

Computing

The statement of subject content in the Computing A Level criteria is brief and contained in the following three paragraphs.

AS specifications in Computing should require learners to develop knowledge and understanding of computer systems and the principles of computing (including programming), and how these are applied to the solution of problems.

A level specifications should require an additional understanding of systematic methods such as the use of algorithms and test strategies, the maintenance of computer systems and the skills associated with documenting solutions.

A level specifications should also require learners to develop further the skills associated with applying this knowledge and understanding to producing computer-based solutions to real problems.

There is an additional section about knowledge and understanding which contains the following mathematical themes.

- *The characteristics of networks and the importance of networking protocols and standards.*
- *Data types, data structures and algorithms.*
- *The organisation and structuring of data and information to facilitate its effective use.*
- *The methods of capturing, selecting, exchanging and managing data to produce information for a particular purpose.*

The specification for the Computing A Level run by OCR includes the following mathematical and statistical content. This illustrates the level of mathematics which may be required in Computing at A Level.

- Number systems: binary, binary-coded decimal, octal and hexadecimal
- Floating point binary numbers
- Representing negative integers
- Adding and subtracting binary numbers
- Structure diagrams
- Producing algorithms to solve problems
- Flow charts
- Implementing algorithms and commenting on efficiency
- Use of iteration in programming
- Use of recursion to solve problems
- Different data types, eg numeric (integer, real), Boolean, character and string
- One and two dimensional arrays for solving problems
- Arithmetic operators including operators for integer division (+, -, *, /, MOD and DIV)
- A range of relational operators, eg =, <, <=, >, >= and <>
- Boolean operators AND, OR and NOT
- Understanding the effects of the precedence of standard operators and the use of parentheses to alter the order of precedence;
- Evaluating expressions containing arithmetic, relational and Boolean operators and parentheses;
- Variables and constants
- Parameters, local and global variables
- Selecting suitable test data
- Searching and sorting algorithms
- Obtaining and analysing data

Applied ICT

Applied ICT is available as a single or a double award. Learners who go beyond a single AS in Applied ICT may have some choice of content.

The AS core includes the following content.

- *Learners learn to find, select, manipulate and communicate information appropriately, and to develop and present this information in a way that is well suited to its purpose and audience.*
They should learn how to manipulate:
 - *numerical data;*
 - *data sets (both large and small);*
 - *graphic images;*
 - *sound and moving images*

Appendix 4

Acknowledgements

MEI would like to thank everyone who has given advice and help that has fed into this review. The list below is by no means complete; discussions with a variety of people have provided food for thought. Many more people have given information to the Integrating Mathematical Problem Solving project as a whole.

Steve Brace, Royal Geographic Society

Charlotte Christie, AQA

Clare Green, Institute of Physics

Vinay Kathotia, Nuffield Foundation

Rachel Lambert-Forsyth, The Society of Biology

Peter Main, Institute of Physics

Hilary Taunton, CIE