Fluency and Problem Solving in Primary Mathematics

MEI's response to the draft primary national curriculum

July 2012

Introduction

MEI has substantial expertise in secondary mathematics and recognises that, because mathematics is a hierarchical subject, the understanding of mathematics and the attitudes to it that students bring from primary school can have a profound effect on their subsequent mathematical progress.

MEI welcomes the level of detail in the draft Primary National Curriculum in Mathematics; primary teachers may not be experts in mathematics and so it is helpful to state clearly what should be taught. However, as currently set out, the draft curriculum could be misinterpreted as just a list of content to be transmitted; it would be helpful to include some structure that makes explicit the connections within mathematics, and with other subjects, to emphasise the importance of the following statement in the introduction to the draft curriculum.

> *… mathematics is a highly inter-connected discipline. Pupils should therefore be taught to practise and then apply their mathematics to a range of problems. They should also be encouraged to make connections across mathematical procedures and concepts to ensure fluency, mathematical reasoning and competence in solving problems. They should also apply their mathematical knowledge in science and other subjects.*

MEI has not at this stage made many comments on the detail of the content of the draft primary curriculum but does consider there is a danger that the development of students' problem solving skills may be neglected in favour of concentrating on teaching them techniques, to the detriment of their mathematical progress. The importance of problem solving is discussed below.

MEI is in agreement with the aims of the draft Primary National Curriculum in Mathematics.

The National Curriculum for mathematics aims to ensure all pupils:

- *become **fluent** in the fundamentals of mathematics so that they are efficient in using and selecting the appropriate written algorithms and mental methods, underpinned by mathematical concepts*
- *can **solve problems by** applying their mathematics to a variety of problems with increasing sophistication, including in unfamiliar contexts and to model real-life scenarios*
- *can **reason mathematically** by following a line of enquiry and develop and present a justification, argument or proof using mathematical language.*

MEI's discussions with employers and with mathematicians and users of mathematics in higher education confirm these are the competences students should develop in order to be able to use mathematics successfully in future life and learning. MEI agrees that it is of the utmost importance that the foundations of all these competences should be laid securely in primary education.

This is further underlined by Sir Michael Wilshaw in his foreword to Ofsted's 'Made to Measure' report (May 2012).[1]

> *It is therefore fundamentally important to ensure that all pupils have the best possible mathematics education. They need to understand the mathematics they learn so that they can be creative in solving problems, as well as being confident and fluent in developing and using the mathematical skills so valued by the world of industry and higher education*.

The importance of problem solving

The Alberta kindergarten to grade 9 Mathematics Curriculum[2] includes the following statement, which illustrates the critical importance of problem solving for improving mathematical fluency.

> *A true problem requires students to use prior learnings in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.*

The New Zealand Mathematics Curriculum[3] expresses a similar philosophy.

> *The solutions to problems which are worth solving seldom involve only one item of mathematical understanding or only one skill. Rather than remembering the single correct method, problem solving requires students to search the information for clues and to make connections to the various pieces of mathematics and other knowledge and skills which they have learned. Such problems encourage thinking rather than mere recall.*

This is supported by Ofsted's 'Made to Measure' report.

> *The emphasis almost all of the schools in the good practice survey placed on pupils using and applying their arithmetic skills to solving a wide range of problems was striking. Diverse opportunities were provided within mathematics, including measures and data handling, and through thematic and cross-curricular work. Pupils' extensive experience of solving problems deepened their understanding and increased their fluency and sense of number.*

The following comment in 'Made to Measure' makes it clear that knowing that problem solving is important is not sufficient to ensure that it is adequately addressed in teaching.

> *Schools were more aware than at the time of the previous survey of the need to improve pupils' problem-solving and investigative skills, but such activities were rarely integral to learning except in the best schools where they were at the heart of learning mathematics. Many teachers continued to struggle to develop skills of using and applying mathematics systematically.*

MEI recognises that primary teachers are rarely mathematics specialists, so it is especially important that they have adequate professional development and resources to enable them to incorporate problem solving into their mathematics teaching. Mathematics is unique as it is a language that can be used to aid problem-solving in all disciplines; it is also important that children learn to solve problems and puzzles within mathematics itself.

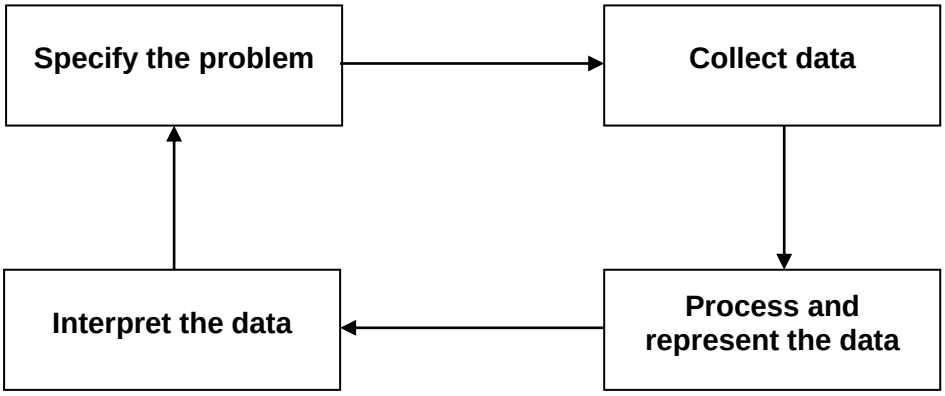
Problem-solving is a fundamental part of study in all disciplines, even at primary level; indeed, this is especially the case at primary level where so much work is genuinely cross-curricular. Cross-curricular problem-solving is often driven by an appreciation of data. MEI welcomes the continued presence of some work on data in the draft curriculum but is disappointed that this is buried in geometry and measures rather than being celebrated as a key and creative component in its own right.

Improving understanding of the problem solving process

The incorporation of the data handling cycle into the KS3 and 4 National Curriculum has emphasised the importance of using statistical techniques as part of a meaningful enquiry. The Royal Statistical Society and Actuarial Profession Report, 'The Future of Statistics in our Schools and Colleges' (2012)[4], recommended that the data handling cycle should also be incorporated at KS2.

> *Recommendation 14: The programme of study for Key Stage 2 should include the data handling cycle, as is currently the case for Key Stages 3 and 4.*

The data handling cycle which is currently used in Key Stages 3 and 4 is reproduced below.



The data handling cycle

As part of a wider commitment to the importance of problem solving and its incorporation into other areas of the curriculum, MEI has done some development work on financial problem solving; a financial problem solving cycle is included in the Appendix. This is a further development of work done for the All Party Parliamentary Group report 'Financial Education & the Curriculum' (2011)[5].

Problem solving strategies

Incorporating a problem solving cycle into the national curriculum is not, of itself, a sufficient means of improving classroom teaching. Fundamental improvements in classroom practice, supported by appropriate professional development and resources, are needed. The cross-curricular nature of primary education should provide many opportunities for problem solving.

Changes to assessment will also be necessary, reflecting the extended work that is inherent in problem solving and the skills we all want students to develop for their future working lives.

MEI recently commissioned an evaluation of a pilot project using a teaching and learning method known as Realistic Mathematics Education (RME), developed at the Freudenthal Institute in the Netherlands; further information about RME can be found in the Freudenthal Institute Wiki[6] .

The philosophy underpinning RME is that students should develop their mathematical understanding by working from contexts that make sense to them. Initially, they devise their own intuitive methods for working on problems but, using a carefully chosen sequence of examples and appropriate teacher interventions, they then generalise and develop a more formal understanding.

The evaluation[7] showed a significant improvement in problem solving.

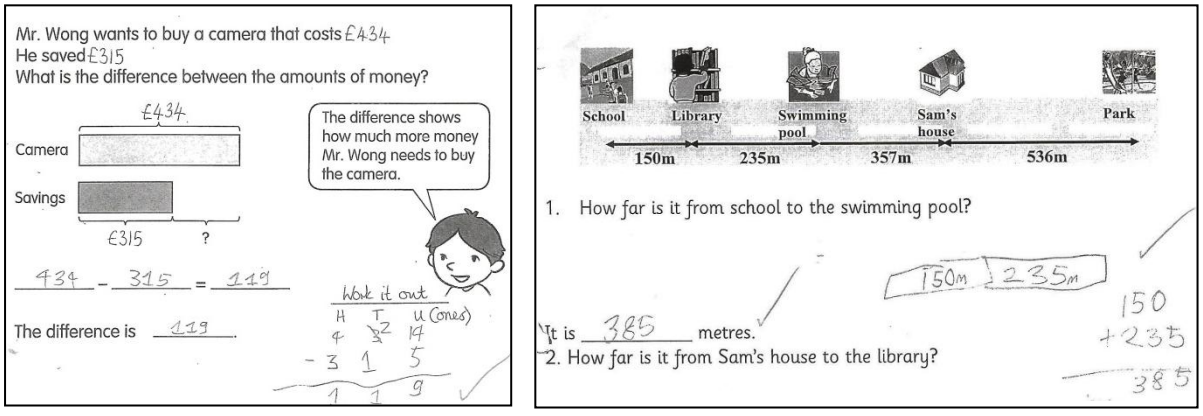
> *Some assessment data from Year 7 pupils from the 2004-06 MiC project were reanalysed using Rasch modelling. This compared achievement and understanding of pupils who had experienced RME with a matched group of pupils who had not. The results indicated those pupils who had experienced RME were not only more likely to solve a problem correctly, but showed considerably more understanding through their ability to explain their strategy.*

Although our experience with RME is at secondary level, results from year 7 students suggest that a teaching approach which encourages problem solving would also be successful at KS2.

ARK Academies have adopted the Mathematics Mastery programme[8], based on teaching materials from Singapore. This programme was described in Ofsted's Good Practice in Primary Mathematics (Nov 2011)[9]

> *One school has recently adopted the Singapore curriculum, which emphasises the consistent use of visual representation to aid conceptual understanding. For instance, 'bar models' are used to represent the relative sizes of quantities and fractional parts. The images below show use of bar models in solving addition and subtraction problems in Year 2.*

Figure 18: Pupils' use of visual representation in solving problems.



There are some similarities between the problem solving strategies used in Singapore text books and models used in teaching RME. The experience of the ARK Academies suggests

that the inclusion of specific problem solving strategies in the school curriculum can improve pupils' understanding.

Conclusion and recommendations

In his letter of 11 June 2012 to Tim Oates about the National Curriculum review[10], Michael Gove made the following statement.

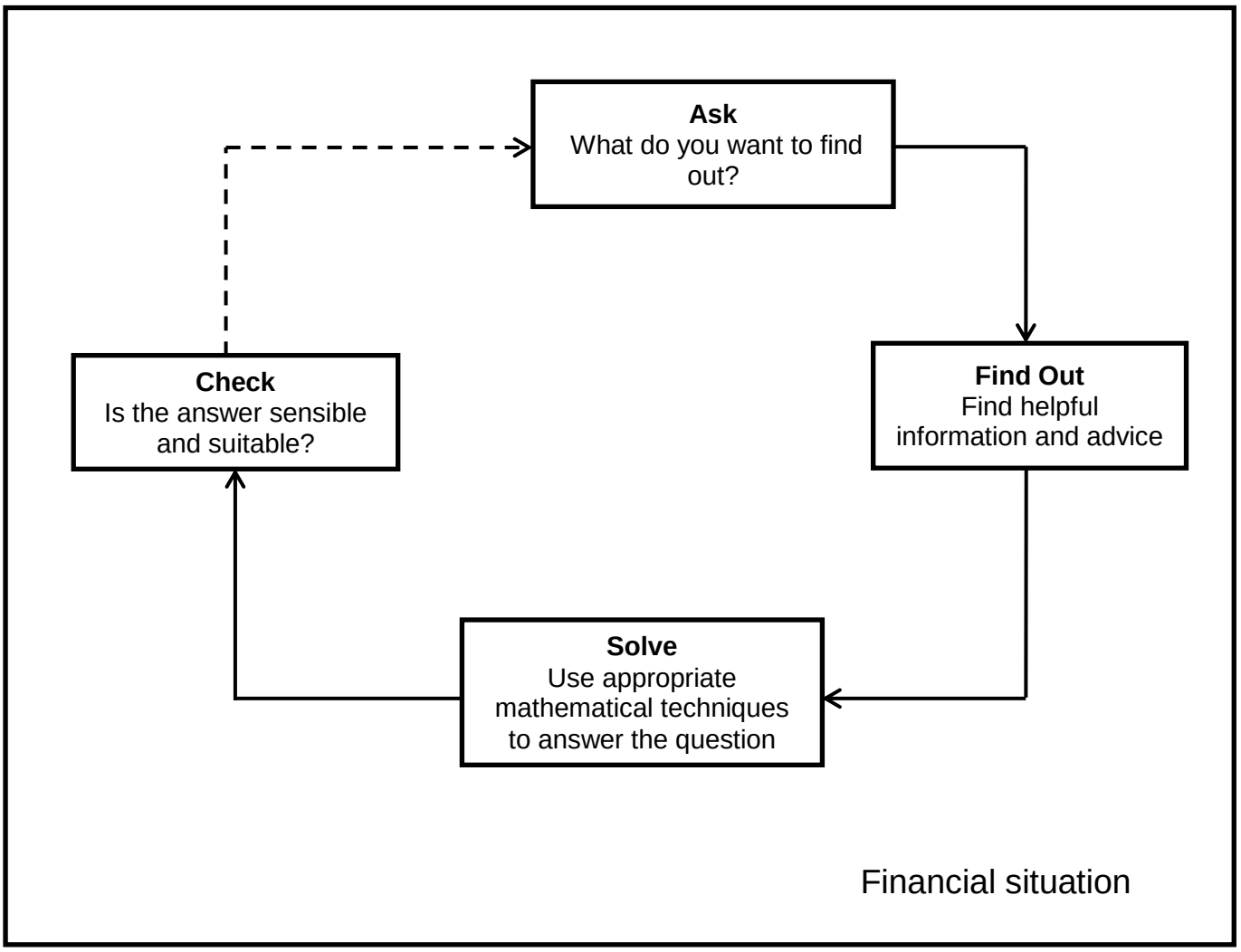
> *And our curriculum changes must provide the gifted teachers we have in our classrooms with both a sense of the higher standards that we know they are driven to reach and the freedom to develop more innovative and effective approaches to teaching.*

MEI agrees and does not believe that a national programme specifying teaching methods would be effective in improving mathematical understanding in schools. However, it is unfortunately the case that many primary teachers do not feel confident in teaching mathematics. We consider that it is essential that suitable resources and professional development are made available to primary teachers to ensure that the good practice found in some schools is disseminated widely. Showing teachers methods which have worked for others will enable them to make effective choices for their own classes. This is in line with the approach taken in Finland, highlighted in the 2010 Ofsted report 'Finnish pupils' success in mathematics'[11].

> *When problem solving was introduced into the Finnish curriculum in 1985, the National Board of Education provided in-service training for teacher trainers and textbook writers and, subsequently, for teachers. Thus, a consistent interpretation of problem solving developed.*

Appendix
Financial Problem Solving at KS2

Simple financial problem solving cycle
Suitable for use with students starting on problem solving



The financial problem solving cycle above is based on Polya's problem solving cycle, described in 'How to solve it' (1945)[12], but the language has been simplified for use at KS2. The cycle can easily be adapted to solve other types of problems with the aid of mathematics.

References

1. Mathematics: Made to Measure, Ofsted, 2012
2. Alberta kindergarten to grade 9 Mathematics Curriculum (page 6)
3. New Zealand Mathematics Curriculum (page 11)
4. The Future of Statistics in our Schools and Colleges
5. Financial Education and the Curriculum
6. Freudenthal Institute Wiki
7. Evaluation Report on the Realistic Mathematics Education Pilot Project
8. Mathematics Mastery
9. Good Practice in Primary Mathematics
10. Letter from Michael Gove to Tim Oates
11. Finnish pupils' success in mathematics
12. How to solve it, G Polya, Penguin, ISBN 0140124993