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Mathematics  
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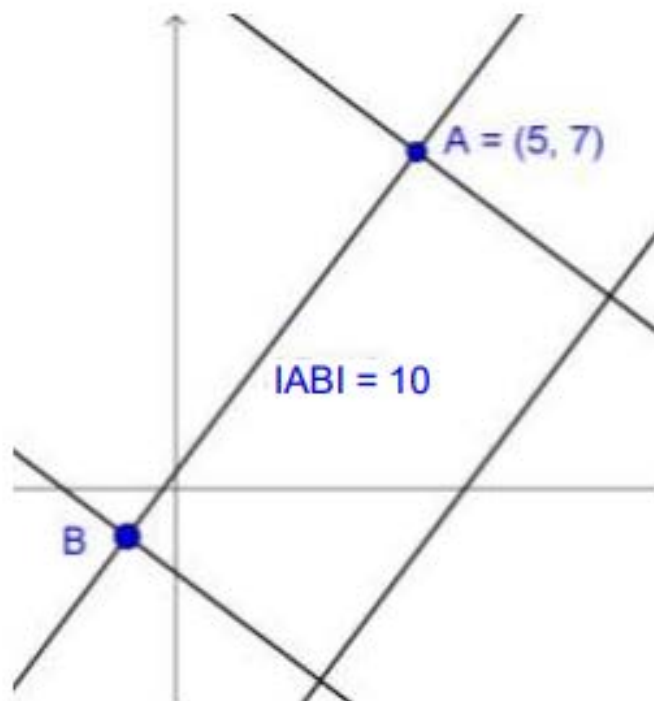
Over 50 years  
at the forefront  
of Mathematics  
Education

**Mathematical Problem Solving**  
A guide for teachers



# Example problems for A level Mathematics

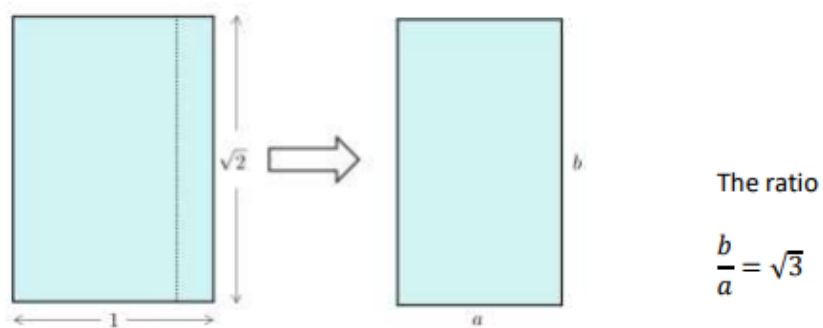
## A level: Example 1



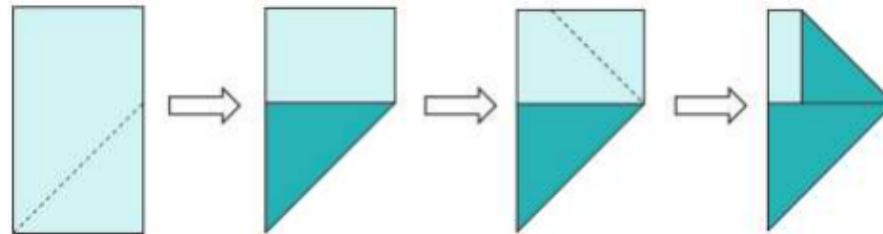
Find the equations of the four straight lines in this diagram.

## A level: Example 3

A piece of paper measures 1 unit by  $\sqrt{2}$  units. A strip of paper is removed so that the dimensions of the paper are  $a$  and  $b$  as shown in the diagram below:



The piece of paper is now folded like this:



**Find the perimeter and area of the final shape.**

## A level: Example 5

### Student instruction sheet

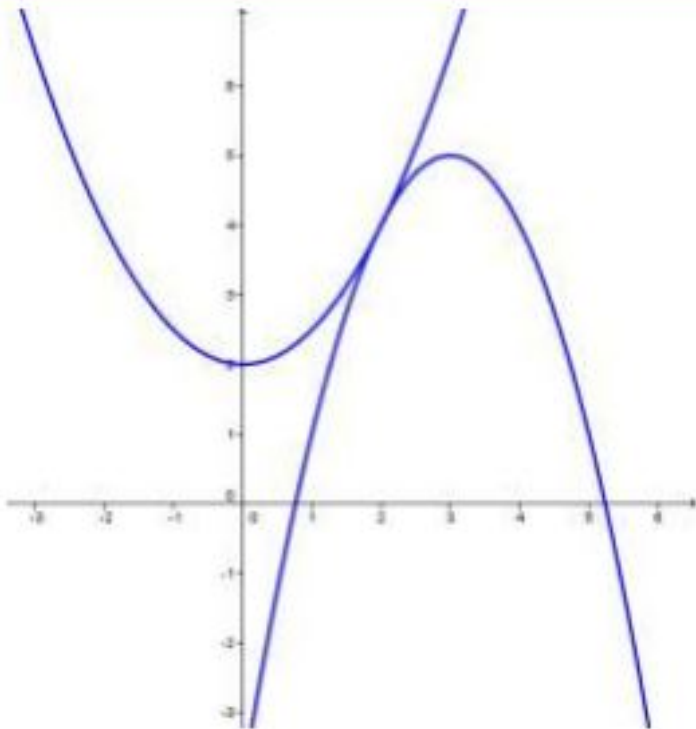
You are going to find a design that has been created using circles.

Some information about the design is given on the cards.

- Shuffle the cards and deal them all out
- Take it in turns to select one piece of information that you think is important and read it to the rest of the group
- You can write something down on the group's answer sheet but you must not show anyone any of your cards (even after you have read them out)
- You can remind the group of any information you have already read out at any stage
- You will need to go round the group more than once to get enough information to sketch the design
- As a group, sketch the design labelling each circle with its equation

What is the fewest number of cards needed to solve the problem?

## A level: Example 9



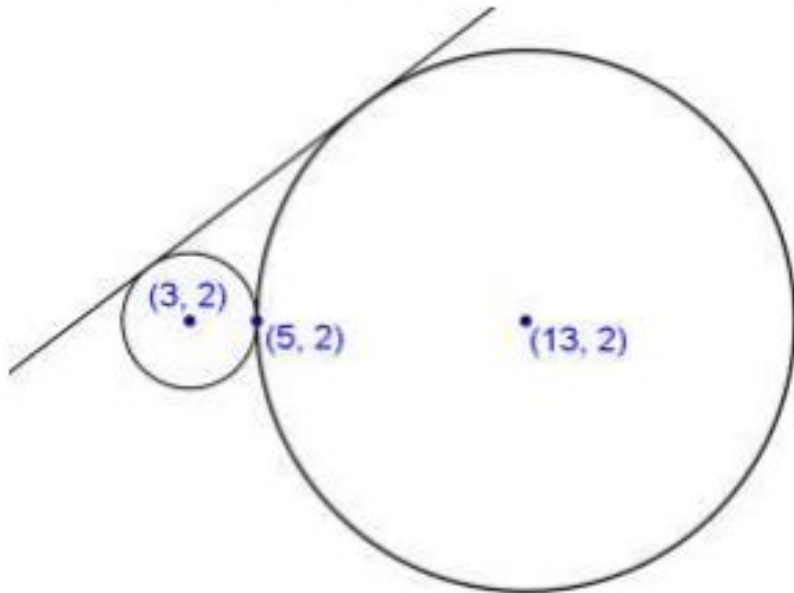
These two quadratic curves have a common tangent at  $x = 2$ .

Given that one curve has a vertex at  $(0,2)$  and the other has a vertex at  $(3,5)$ , find the equation of each curve and the equation of the common tangent.

## A level: Example 11

### Example 11

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The diagram shows two circles and a straight line that is a tangent to both circles.

Find the equation of the line.

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## A level: Example 15

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Without using a calculator, show that

$$\sqrt{3} - \sqrt{2} = \sqrt{5 - 2\sqrt{6}}$$

Find better ways to write

(i)  $\sqrt{12 - 2\sqrt{35}}$

(ii)  $\sqrt{13 + 2\sqrt{42}}$

(iii)  $\sqrt{21 + 6\sqrt{10}}$

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## A level: Example 16

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Write down the first five terms of

$$\sum_{r=1}^{10} r^2$$

Write down the first five terms of

$$\sum_{r=1}^{10} (r-1)^2$$

Use your answers to help you find the value of

$$\sum_{r=1}^{10} r^2 - \sum_{r=1}^{10} (r-1)^2$$

Find an expression in terms of  $n$  for the value of

$$\sum_{r=1}^n r^2 - \sum_{r=1}^n (r-1)^2$$

Find an expression in terms of  $m$  and  $n$  (where  $n > m$ ) for the value of

$$\sum_{r=m}^n r^2 - \sum_{r=m}^n (r-1)^2$$

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## A level: Example 18

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Find the values of  $a$  such that the turning point (i.e. the vertex) of the parabola  $y = x^2 + 2ax + 1$  is closest to the origin.

What is the locus of the turning point as  $a$  varies?

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## A level: Example 19

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A geometric progression has first term  $a$  and common ratio  $r$ , and the terms are all different and  $a \neq 0$ . The first, second and fourth terms of the geometric progression form three consecutive terms of an arithmetic progression. Given that the geometric progression converges and has a sum to infinity of  $3 + \sqrt{5}$ , find the exact values of  $a$  and  $r$ .

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## A level: Example 20

In this question you must show detailed reasoning.

Fig. 1 shows the circle with equation  $(x - 3)^2 + (y - 4)^2 = 25$ . The point  $C$  is the centre of the circle.

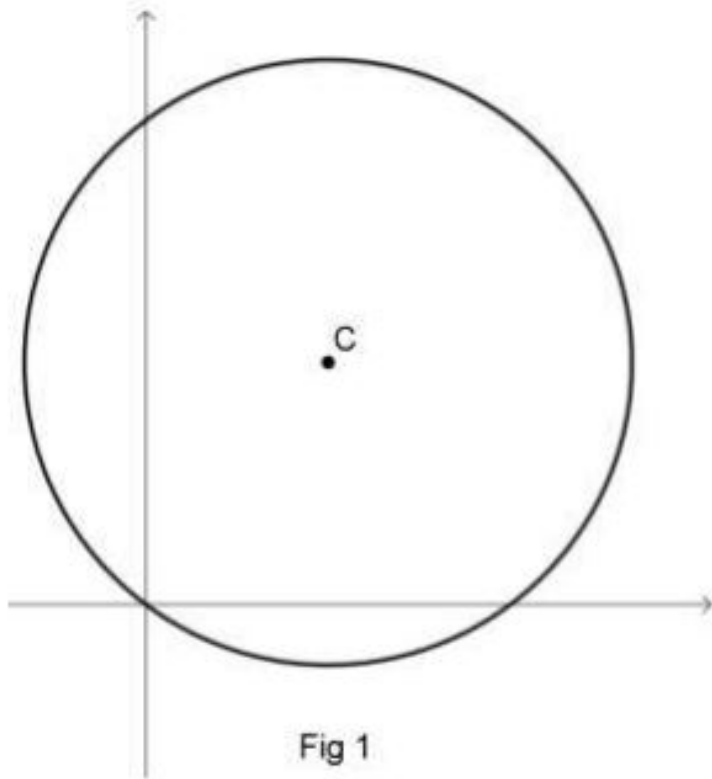


Fig 1

Find the area of the part of the circle that is below the  $x$ -axis.

# About MEI

- Registered charity committed to improving mathematics education
- Independent UK curriculum development body
- We offer continuing professional development courses, provide specialist tuition for students and work with employers to enhance mathematical skills in the workplace
- We also pioneer the development of innovative teaching and learning resources

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