

# Mathematical Problem Solving

## AS/A Level example

### Example 16

The class have been shown how to use sigma notation for series and are practising its use. The problem below is used as part of the exercise.

#### Understand and use sigma notation for sums of series

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Write down the first five terms of

$$\sum_{r=1}^{10} r^2$$

Write down the first five terms of

$$\sum_{r=1}^{10} (r-1)^2$$

Use your answers to help you find the value of

$$\sum_{r=1}^{10} r^2 - \sum_{r=1}^{10} (r-1)^2$$

Find an expression in terms of  $n$  for the value of

$$\sum_{r=1}^n r^2 - \sum_{r=1}^n (r-1)^2$$

Find an expression in terms of  $m$  and  $n$  (where  $n > m$ ) for the value of

$$\sum_{r=m}^n r^2 - \sum_{r=m}^n (r-1)^2$$

This problem starts off with two questions that could have been part of the exercise. It then hints at a useful technique from Further Mathematics (the method of differences) before getting the students to generalise.

The procedures from this guide can be applied:

- A question and answer session
  - ◆ How do the answers to the first two parts help?
  - ◆ Are we sure we can always generalise?
- Engagement with the problem
  - ◆ The students try to work it out
- Student review
  - ◆ A check to see how and what they are doing
  - ◆ What is different about the last part?
  - ◆ Why does  $n$  have to be greater than  $m$ ?
  - ◆ How can we adapt our method from the previous parts?
- Reflection
  - ◆ Do you think there are other example that may behave in the same way?

In this example it is likely that there will be little variation in the methods used. The reflection stage could be used to prompt some thought about series in which the same effect could be achieved allowing for some deeper, creative thought.