Mathematical Problem Solving
AS/A Level example

Example 3

The class are practising their skills with manipulating surds.

Use and manipulate surds, including rationalising the denominator

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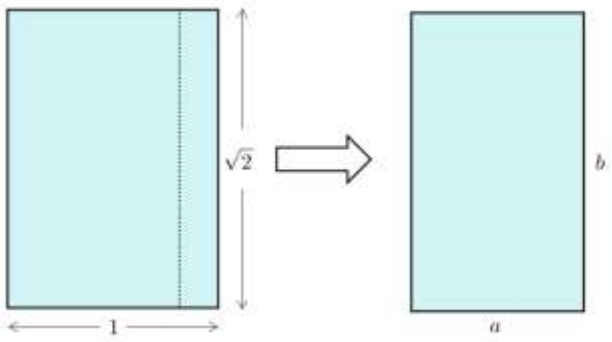
The teacher presents the class with this problem on the board.

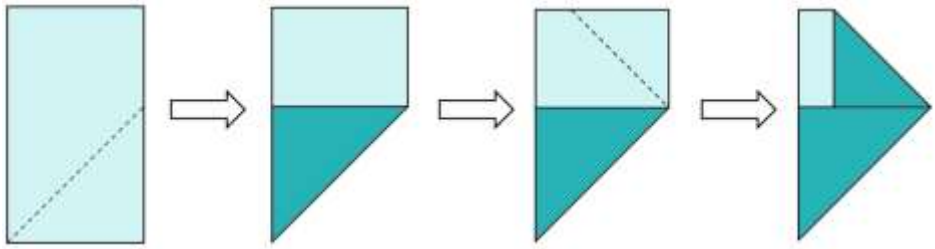
A piece of paper measures 1 unit by $\sqrt{2}$ units. A strip of paper is removed so that the dimensions of the paper are a and b as shown in the diagram below:



The ratio

$$\frac{b}{a} = \sqrt{3}$$

The piece of paper is now folded like this:



Find the perimeter and area of the final shape.

As was the case with the *incomplete* problems, it is important that the teacher makes it clear that the task also develops problem-solving skills. This time, the students need to be made aware that most of the required information is contained within the diagrams in the question and that they are going to go through the same process of asking questions to find that information. Additional information is needed to solve the problem but this is what the problem is testing. The main difference to the questioning this time is that the students will have to come up with the answers to those questions themselves.

The procedure is the same:

The students should be given two minutes time to consider the problem without writing anything down. They should then spend two more minutes writing down as many questions as they can that would give key information for solving the problem. Students should then spend two more minutes comparing their questions with those of one or two other students. They need to be reminded to say why they think their question will reveal some important information.

The students should hopefully come up with some questions like these:

- Does the paper remain the same height?
- What is the width of the strip that is cut off?
- What are the new dimensions of the paper?
- What is the length of the diagonal line made by the first fold?
- How much remains of edge b?
- What is the length of the diagonal line made by the second fold?
- How much remains of edge a?

There may well be other valid questions. For this example the teacher should encourage the students to answer their own questions.

The teacher should encourage and model the use of accurate mathematical language wherever possible.

The responses below are examples of what may be said. It may not be necessary for the teacher to say that much. Answers may be forthcoming from the group discussion.

Working through the questions in order:

Question	Answer
Does the paper remain the same height?	Look at the diagram. How is the strip cut off? Will this affect the height of the paper? *No!* So it does remain the same height.
What is the width of the strip that is cut off?	You are going to need to calculate this. How can you start to do this? *Give the width a name e.g.* x. What are a and b in terms of your unknown and $\sqrt{2}$? a *is* $1 - x$ b *is still* $\sqrt{2}$ How can you use the information in the problem to find the values of x? $\frac{b}{a} = \sqrt{3}$ *we can use that.*
What are the new dimensions of the paper?	You now have x, you tell me!
What is the length of the diagonal line made by the first fold?	What sorts of shapes are you getting? What do you know about these shapes? Are there any shortcuts you can use? *The teacher should encourage the students to be efficient. It is not necessary to use Pythagoras's rule as they should know something about* $45°$ *right angled triangles.*
How much remains of edge b?	How much of edge b is removed by folding along the line?

Question	Answer
What is the length of the diagonal line made by the second fold?	You've already done something like this for the first fold. How can you use the same idea for the second fold?
How much remains of edge a?	How much of edge a is removed by folding along the line?