Mathematical Problem Solving
GCSE example

Example 6

A higher tier group have been studying probability including mutually exclusive events, independent and dependent combined events and simple expected values from probabilities.

- **relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale**
- **apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one**
- **enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams**
- **construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities**
- **calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions**

DfE: Mathematics GCE AS and A level subject content July 2014

The teacher presents the class with this mystery task talking the students through the process. The students are placed into groups of 3, 4 or 5 and each group is provided with a set of the cards, some lined paper and a sheet of A3 or flipchart paper.

As this is quite a challenging task, the teacher should talk through the instructions carefully pointing out that the students will need to find out how many people there are in each age group, how many of each group will have already been vaccinated and not need treatment, how many will be tested for flu and how many will need treatment. This should focus the students' minds on the best order to give up their information and what they should be doing with it.

Student instructions

A new strain of flu virus is sweeping across the country.

As the management team of a small healthcare practice, you are in charge of ordering anti-viral medicine.

How many boxes of anti-viral medicine will you need to order?

You do not wish to order too many or too few.

Use the clues on the cards to calculate a sensible estimate.

- Shuffle the cards and deal them all out
- Take it in turns to select one piece of information that you think is important and read it to the rest of the group
- You can write something down on the group's answer sheet but you must not show anyone any of your cards (even after you have read them out)
- You can remind the group of any information you have already read out at any stage
- You will need to go round the group more than once to get enough information to solve the problem
- As a group, try to find a sensible number of boxes of antiviral medicine to order
- Show your calculations and your answer on your group answer sheet.

What is the fewest number of cards needed to solve the problem?

The student cards are on the next page.

There are 1913 people registered with the practice.	An estimated 1% of people registered will be pregnant women.	Children under 6 require 1 box of antiviral medicine.	An estimated 45% of unvaccinated over 64s will exhibit symptoms of flu.
Pregnant women, over 64's and under 6's are considered to be the "at risk" groups.	17% of people registered are over 64.	An estimated 85% of the "low risk" group that have symptoms of flu will choose not to be tested.	Flu medicine can be ordered in sets of 10 boxes.
Adults and children aged 6 and over require 2 boxes of antiviral medicine.	At risk groups are tested for flu as soon as they exhibit any symptoms.	8% of people registered are under 6.	The estimated probability of a person with the symptoms actually having flu is 0.7.
56% of the over 64 age group had a flu vaccination at the start of the winter.	People in the "low risk" group can choose to be tested and receive antiviral medicine if they wish.	An estimated 30% of pregnant women will exhibit symptoms of flu.	People who are aged from 6 to 64 and aren't pregnant are considered to be "low risk."
An estimated 30% of the "low risk" age group will exhibit symptoms of flu.	An estimated 55% of under 6s will exhibit symptoms of flu.	People who have received the flu vaccine will not display any symptoms and will have virtually no chance of contracting this strain of flu.	People with no symptoms are not tested for flu.

Commentary

This commentary follows the discussions of a group of 4 students attempting to solve the problem. Their comments have been paraphrased to make them clearer for the reader.

The initial distribution of the cards amongst the 4 students was:

Player	Cards				
1	17% of people registered are over 64.	Children under 6 require 1 box of antiviral medicine.	An estimated 30% of pregnant women will exhibit symptoms of flu.	56% of the over 64 age group had a flu vaccination at the start of the winter.	An estimated 55% of under 6s will exhibit symptoms of flu.
2	Flu medicine can be ordered in sets of 10 boxes.	People with no symptoms are not tested for flu.	An estimated 85% of the "low risk" group that have symptoms of flu will choose not to be tested.	An estimated 1% of people registered will be pregnant women.	At risk groups are tested for flu as soon as they exhibit any symptoms.
3	An estimated 45% of unvaccinated over 64s will exhibit symptoms of flu.	People who are aged from 6 to 64 and aren't pregnant are considered to be "low risk."	There are 1913 people registered with the practice.	People in the "low risk" group can choose to be tested and receive antiviral medicine if they wish.	The estimated probability of a person with the symptoms actually having flu is 0.7.
4	8% of people registered are under 6.	Adults and children aged 6 and over require 2 boxes of antiviral medicine.	People who have received the flu vaccine will not display any symptoms and will have virtually no chance of contracting this strain of flu.	Pregnant women, over 64's and under 6's are considered to be the "at risk" groups.	An estimated 30% of the "low risk" age group will exhibit symptoms of flu.

The students' comments and notations are detailed below.

Player	Card played	Discussion and calculation	Output
1	56% of the over 64 age group had a flu vaccination at the start of the winter.	I haven't really got a good starting card but when we have some numbers, we can calculate something with this. At least we know one of the groups.	Writes "over 64 56% vacc." on paper.
2	An estimated 1% of people registered will be pregnant women.	I haven't got a good starting card either but this must be another one of the groups.	Writes "pregnant women 1% registered" on paper.
3	There are 1913 people registered with the practice.	This is a good one. We can find out how many of these are pregnant women. 1% of 1913 is 19.13. There must be some rounding going on, let's call it 19.	Writes "19" next to "pregnant women."
4	8% of people registered are under 6.	This must be another group. 8% of 1913 is $19.13 \times 8 = 153.04$. Let's say 153.	Writes "153 under 6."
1	17% of people registered are over 64.	We can work out how many of these there are and how many have been vaccinated. 17% of 1913 is $19.13 \times 17 = 325.21$. Let's call this 325. 56% of them are vaccinated. That's 0.56×325 <<reaches for a calculator>> $= 182$	Crosses out "over 64 56% vacc." And writes "Over 65: 325, 182 vacc."

Player	Card played	Discussion and calculation	Output
2	An estimated 85% of the "low risk" group that have symptoms of flu will choose not to be tested.	I still don't have anything that good. I suppose we can use this later.	Writes "85% low risk not tested" (ignoring the fact that this is only for those with symptoms).
3	An estimated 45% of unvaccinated over 64s will exhibit symptoms of flu.	We need to know how many over 64s are unvaccinated. $325 - 182 = 143$ 0.45×143 <<uses calculator>> $= 64.35$ we'll call it 64.	Writes "64 over 64s have symptoms"
4	Pregnant women, over 64's and under 6's are considered to be the "at risk" groups.	We can group them together now. We can find out how many are low risk too. $19 + 325 + 153 = 497$ $1913 - 497 = 1416$ <<all done using a calculator>> We know how many of these won't be tested. 3 interjects – I should have said that's the ones with symptoms. We don't know how many of them there are yet.	Writes "High risk" as a title and then "Over 64: 325, 64 sympt." followed by "pregnant: 19" and "under 6: 153" Writes "Low risk: 1416"
1	An estimated 30% of pregnant women will exhibit symptoms of flu.	We can do something with this. $0.3 \times 19 = 5.7$ we'll call this 6.	Adds ", 6 sympt." to "pregnant: 19."

Player	Card played	Discussion and calculation	Output
2	At risk groups are tested for flu as soon as they exhibit any symptoms.	At least I know who's going to be tested!	
3	The estimated probability of a person with the symptoms actually having flu is 0.7.	We can do a lot with this. We can find out who's actually got flu. $0.7 \times 64 = 44.8$ - let's say 45 over 64s have flu. $0.7 \times 6 = 4.2$ – and 4 pregnant women have flu.	Writes ", 45 flu" after "Over 64: 325, 64 sympt." and ", 4 flu after "pregnant: 19, 6 sympt."
4	An estimated 30% of the "low risk" age group will exhibit symptoms of flu.	We can do the numbers for this now. <<group discuss calculations>> $0.3 \times 1416 = 424.8$ – call it 425. 85% of them won't be tested so 15% will be tested. $0.15 \times 425 = 63.75$ – call this 64. $0.7 \times 64 = 44.8$ – call this 45.	Writes ", 45 flu" after "Low risk: 1416"
1	An estimated 55% of under 6s will exhibit symptoms of flu.	So that's $0.55 \times 153 = 84.15$ – call that 84 $0.7 \times 84 = 58.8$ – call this 59.	Writes ", 59 flu" after "under 6: 153"
2	Flu medicine can be ordered in sets of 10 boxes.	We'll have to work out how much to order so this will be useful in a bit.	Writes "medicine in 10s"

Player	Card played	Discussion and calculation	Output
3	People in the "low risk" group can choose to be tested and receive antiviral medicine if they wish.	I've used up all of my good cards so I don't have anything helpful left.	
4	Adults and children aged 6 and over require 2 boxes of antiviral medicine.	This should let us find the number of boxes we need. We have $45 + 4 + 45 = 94$ for 6 and over and 59 under 6. That's $94 \times 2 = 188$ boxes for the adults	Writes "188 for over 6s"
1	Children under 6 require 1 box of antiviral medicine.	We could have guessed this but now we know we need $188 + 59 = 247$ boxes. They're sold in 10s so we'd need to order 250 boxes.	Writes "250 boxes" and circles it.

This group only just finished the main activity and did not have time to consider the minimum number of cards required to solve the problem.