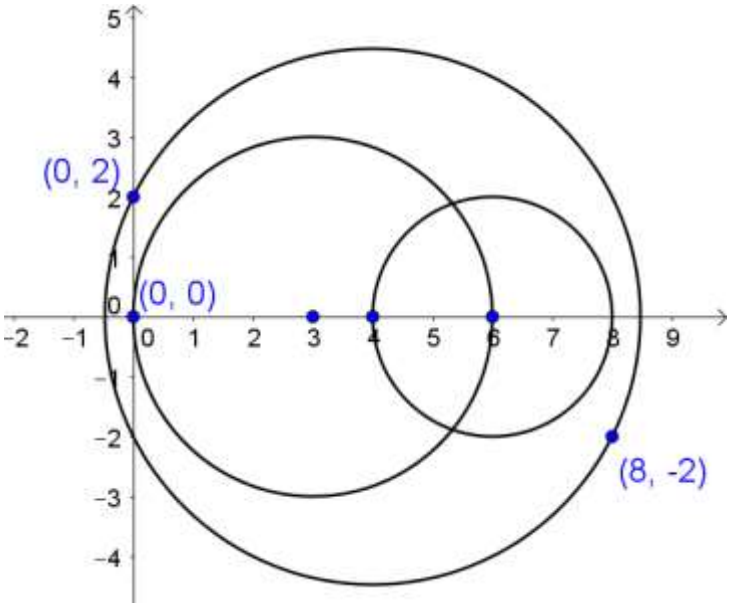
Solution to example 5

Cards		
The second largest circle passes through the origin.	All three circles have their centres on the positive x axis.	The area of the largest circle is 5 times the area of the smallest circle.
The design consists of three circles.	The largest circle passes through the point $(0,2)$.	The design has reflection symmetry.
The centres of the two smaller circles are three units apart.	The two smaller circles overlap.	The point $(5,0)$ is inside all three circles.
The two smaller circles are contained within the largest circle.	The largest circle passes through the point $(8,-2)$.	The smallest circle passes through the centre of the largest circle.
The second smallest circle passes through the centre of the smallest circle.	The ratio of the radii of the two smaller circles is $2:3$	The areas of the three circles are in the ratio $4:9:20$

Diagram



Key information and calculations

There are a number of ways to find the solution to this problem using the information on the cards. One way is shown below but there are others.

Using the cards

- All three circles have their centres on the positive x axis
- The largest circle passes through the point $(0,2)$
- The largest circle passes through the point $(8,-2)$

Let the centre of the largest circle be at $(c,0)$

The distance from $(0,2)$ and $(8,-2)$ to $(c,0)$ will be the same as they are both radii of the largest circle.

So $\sqrt{(c-0)^2+(0-2)^2}=\sqrt{(c-8)^2+(0+2)^2}$

$c^2+4=(c-8)^2+4$

$c^2=(c-8)^2$

$c^2=c^2-16c+64$

$16c=64$

$c=4$

The centre of the largest circle is at $(4,0)$ and it has a radius of $\sqrt{c^2+4}=\sqrt{20}$.

It therefore has the equation $(x-4)^2+y^2=20$

Using the cards

The design consists of three circles

The areas of the three circles are in the ratio $4:9:20$

The second largest circle passes through the origin

The second smallest circle passes through the centre of the smallest circle

The area of the largest circle is $\pi \times \left(\sqrt{20}\right)^2 = 20\pi$

The areas are in the ratio $4:9:20$ means that the middle circle has area 9π and the smallest has area 4π square units.

The radius of the smallest circle is therefore 2 units and the radius of the middle circle is 3 units.

If the middle circle has its centre on the positive x axis, passes through the origin and has a radius of 3 units, its centre must be at $(3,0)$.

The middle circle therefore has equation $(x-3)^2+y^2=9$.

If the middle circle passes through the centre of the smallest circle and the smallest circle has its centre on the positive x axis, it must have a centre at $(6,0)$. Note: the other possibility is $(0,0)$ but that is not on the **positive** x axis, it is at $x=0$.

So, the smallest circle has centre $(6,0)$ and radius 2.

Its equation is $(x-6)^2+y^2=4$

Using these calculations, only 7 cards were required.