Equations and Inequalities (AS)

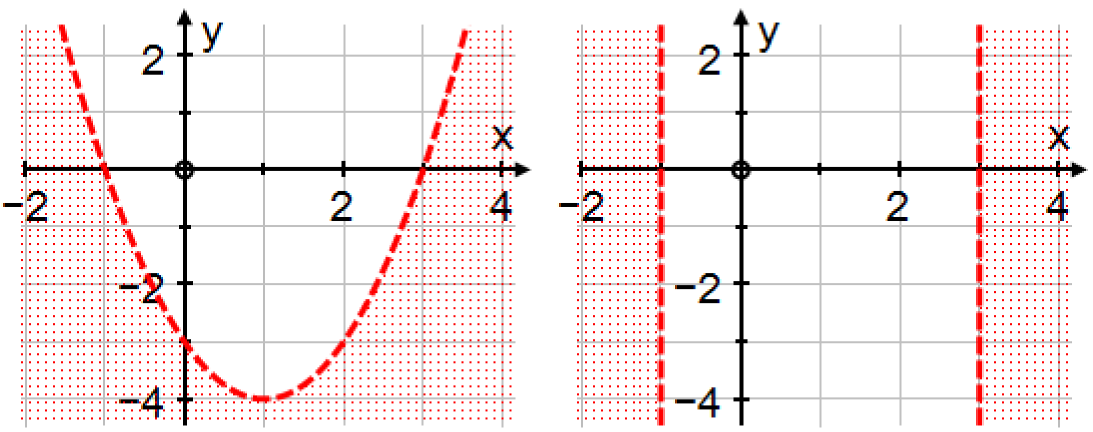
B4	Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation
B5	Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions Express solutions through correct use of ‘and’ and ‘or’, or through set notation Represent linear and quadratic inequalities such as $y > x + 1$ and $y > ax^2 + bx + c$ graphically

Commentary

From GCSE, students may have several methods for solving simultaneous equations. Given a free choice of method, what aspects of the two equations will indicate that one method is more straightforward than another?

When solving $x + y = 5, x - y = 3$ by elimination students might add the equations together to get $2x = 8$. What is happening here at a graphical level? Are the lines being ‘added’; is it possible to add lines together? It might be helpful to think in terms of a specific coordinate pair (a, b) satisfying both equations; then it is unknowns a and b, rather than variables x and y, that are being manipulated.

Consider the differences between the graphical representations below, both related to the quadratic $(x+1)(x-3)$. The left-hand one shows the region $y > (x+1)(x-3)$ and the right-hand one shows the region $-1 < x < 3$ which satisfies $(x+1)(x-3) > 0$.



Students could be asked questions such as, ‘What is each representation showing?’, ‘What might the questions be that generate these as answers?’, and ‘What difficulties might students have in understanding the differences?’

Sample MEI resource

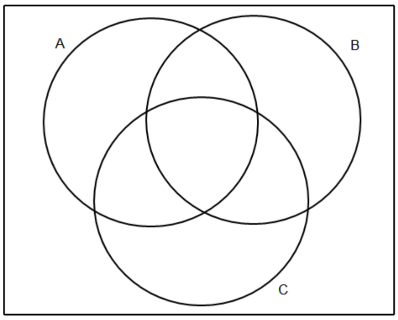
'Categorising quadratic inequalities' (which can be found at http://integralmaths.org/sow-resources.php) is designed to encourage students to become fluent with quadratic inequalities. Once they have completed this task they might be encouraged to make up their own categories for a Venn diagram.

Categorising quadratic inequalities

A: The inequality is satisfied by $x = 2$.

B: The solution is given by $a < x < b$ for integers a and b.

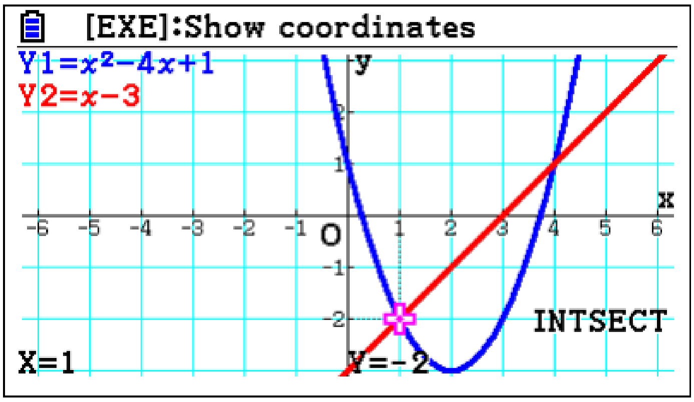
C: The inequality is satisfied by $x = 4$.



Effective use of technology

'Intersection of a line and a curve' (which can be found at www.mei.org.uk/integrating-technology) is designed to support students in using graphing calculators with linear and quadratic simultaneous equations. It comes with questions and further tasks.

1. Add a new Graphs screen: [MENU] [5]
2. Add a quadratic function as Y1, e.g. **Y1=x²–4x+1 :** [X,θ,T] [x^2] [−] [4] [X,θ,T] [+] [1] [EXE]
3. Add a line as Y2, e.g. **Y2=x–3** : [X,θ,T] [−] [3] [EXE]
4. Plot the curves: [F6]
5. Find the points of intersection of the line and the curve: [F5] [F5]



Equations and inequalities (AS)	Time allocation:

Pre-requisites

- GCSE: Linear and quadratic inequalities
- GCSE: Simultaneous equations
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Links with other topics

- Coordinate geometry: intersection of lines and circles
- Calculus: Tangents and repeated roots
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Questions and prompts for mathematical thinking

- What questions could be used to tease out why multiplying an inequality by a negative number means you have to reverse the inequality?
- Change one coefficient in the equations $y = x^2 - 2x + 3,\quad y = x + 2$ so that no value of x satisfies both.
- When $x = \frac{1}{3}$, $x^3 < x^2 < x < 1 - x < \frac{1}{x}$. What happens to the order of inequalities for other values of x?
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Opportunities for proof

- Prove that multiplying an inequality throughout by -1 is the same as reversing the sign.
- Prove that the product of the values of x satisfying both $y = x^2 + x - 2$ and $y = mx$ is -2 for every value of m
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Common errors

- If x is between -2 and -5 writing the inequality as $-2 < x < -5$ rather than $-5 < x < -2$
- Trying to combine two separate inequalities as a single expression, for example writing $x < -4$ and $x > 3$ as $-4 > x > 3$
- Thinking that $(x-4)(x+2) > 0$ means $(x-4) > 0$ and $(x+2) > 0$
- Believing that $(x+3)^2 < 16$ has the same set of solutions as $(x+3) < 4$
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