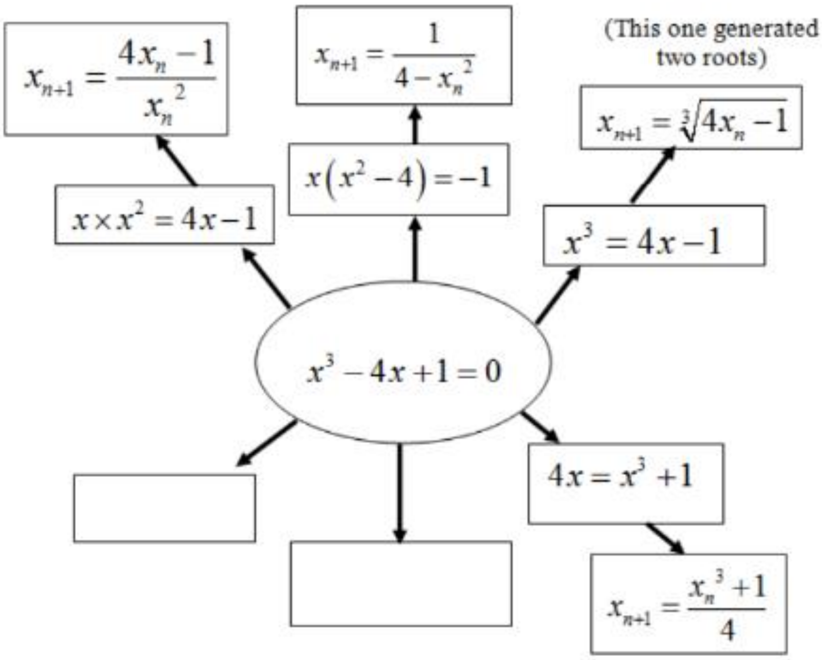
Investigating Iterative Formulae

Lead the class through two or three rearrangements of the initial equation, $x^3 - 4x + 1 = 0$ explaining that you're hoping to find an iterative formula that would generate all three roots. Make some suggestions, if the students can't prompt you for any themselves.



In small groups, the students should use one of these iterative formulae with different starting values and produce posters (using A1 paper, ideally) on which they record their work. They can include sketches of graphs and related diagrams, calculations, conclusions they make, and the reasoning they apply as they tackle the problem.

Questions or points to highlight as you circulate among the groups of students

Why do some iterative formulae lead to numbers that diverge?

Can you find another rearrangement different from the ones we've done?

How quickly did the sequence converge? To what extent does it depend on x_0?

For a single iterative formula generating two roots, where is the critical point for x_0 where it flips over from one root to another?

Are there any initial values that cause problems?

If your sequence converges, this means that both x_n and x_{n+1} are getting close to some number. Call this number A. Then $x_{n+1} = \sqrt[3]{4x_n - 1}$ leads to $A = \sqrt[3]{4A - 1} \Rightarrow A^3 = 4A - 1 \Rightarrow$ $A^3 - 4A + 1 = 0$ and so you can see that the number A is a root of the equation

$x^3 - 4x + 1 = 0.$

Try this one: $x^4 - 6x^2 + 4 = 0$. Alternatively, if they've studied exponentials you could set questions like $e^x - x^2 - 2 = 0$ and $x^2 = \ln(x + 1)$